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ABSTRACT

This study compared the effectiveness of 10 methods of determining the number of factors to retain in exploratory common factor analysis. The 10 methods included the Kaiser rule and a modified Kaiser criterion, 3 variations of parallel analysis, 4 regression-based variations of the scree procedure, and the minimum average partial procedure. The performance of these procedures was evaluated based on the average number of factors retained by each method, the proportion of samples retaining the same number of factors retained by each method, the proportion of samples retaining the same number of factors as the true number of factors in the population, and the proportion of samples retaining the same number of factors when a particular rule of thumb is applied to the population. The performance of the 10 procedures was investigated using Monte Carlo methods in which random samples were generated under known and controlled population conditions. Results clearly suggest that both the choice of method and the design of the factor analytic study play crucial roles in retaining the correct number of common factors. In terms of overall accuracy across the conditions examined in this study, one of the parallel analysis approaches (Montanelli and Humphreys, 1976) provided the largest proportion of samples retaining the correct value. Conditions under which other approaches may work well are discussed. (Contains 16 tables, 18 figures, 25 charts, and 44 references.) (SLD)

Factor Retention in Exploratory Factor Analysis: A Comparison of Alternative Methods

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Factor Retention in Exploratory Factor Analysis:
A Comparison of Alternative Methods

This study extends previous investigations of the effectiveness of various procedures for determining the number of factors to retain in exploratory common factor analysis. In our previous Monte Carlo study on the quality of factor analytic solutions (Ferron, Kromrey, Hogarty, Hines, & Mumford, 2002), we found that decision rules used to make a determination of the number of factors to retain, in most cases, yielded underestimates of the known number of factors in the population. Zwick and Velicer (1986) suggest that the determination of the number of factors or components to retain is likely to be the most important decision that the researcher makes in the conduct of a factor analytic study. Researchers conducting exploratory factor analyses in most instances do not know the true number of factors that are expected to underlie the data, and may make decisions resulting in the retention of too few or too many factors. Since the number-of -factors decision is made prior to the factor rotation stage, it subsequently impacts the results of the factor analysis, such as rotated factor patterns, factor score estimates, and the interpretability of the factors. Consequently, several researchers (e.g., Rummel, 1970; Fava & Velicer, 1996; Wood, Tataryn, & Gorsuch, 1996) have emphasized the importance of extracting the correct number of factors for rotation. As Turner (1998) and others note, the interpretation of the factors is based on the assumption that the researcher has extracted the correct number of factors.

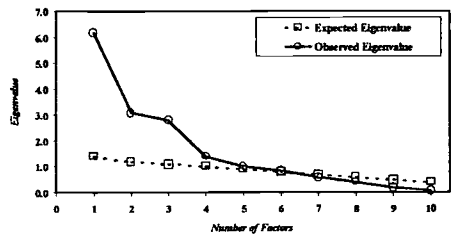
The problems of underfactoring (i.e., extracting too few factors) and overfactoring (i.e., extracting too many factors) have been addressed in the literature (see for example, Crawford, 1975; Fava & Velicer, 1992, 1996; Gorsuch, 1983; MacCallum, Widaman, Preacher & Hong, 2001; Mosier, 1939; Turner, 1998; Wood, Tataryn & Gorusch, 1996; Zwick & Velicer, 1986). In the case of underfactoring, retaining too few factors for inclusion in the rotation phase can result in the loss of potentially important information (e.g., a substantive factor or factors) in the factor solution. With underfactoring, it has been argued, the true factors in a data set cannot be accurately portrayed (Cattell, 1958; Comrey, 1978; Fava & Velicer, 1996). Wood et al.

(1996) note that when underfactoring occurs, the estimated factors are likely to contain considerable error. In their investigation of the consequences of underfactoring in both maximum likelihood factor analysis and principal component analysis, Fava and Velicer (1996) found "severe degradation of factor score estimates" with underfactoring. It was noted that the principal component score degraded less rapidly than the factor score within methods.

Most researchers concur that overfactoring is less a problem than underfactoring. Fava and Velicer (1992) and others offer two theoretical justifications that support this point of view. The first, they posit, is based on the fact that each subsequent factor extracted accounts for less variance than the factor extracted prior to it. The second relates to the notion that if too many factors are retained, after rotation, it is relatively easy to discard trivial factors without changing the substantive factors. This notwithstanding, there is evidence to suggest that overfactoring is none-the-less quite problematic and should be avoided. With overfactoring, the resultant factor solution may include factors that are not interpretable or unlikely to replicate (Zwick & Velicer, 1986). It has been suggested that overfactoring may result in factor splitting at the rotation phase, or when rotating too many oblique factors, high interfactor correlations may result or the factor space may collapse (Crawford, 1975). Gorsuch (1983) warns that the extraction of too many factors may cause a common factor to be missed. In addition, several studies lend support to the notion that overfactoring may result in change of the overall factor structure (see for example, Keil & Wrigley, 1960; Howard & Gordon, 1963; Dingman, Miller, & Eyman, 1964).

Wood et al. (1996) found that when overfactoring occurs, the estimated loadings for true factors usually contain less error than in the case of underfactoring. Based on the findings of their study, these researchers suggest that overfactoring is preferable to underfactoring provided that factor splitting is prevented and false factors are eventually eliminated (these authors advance methods for handling these two conditions). Fava and Velicer (1996) also concluded from their series of studies, that underfactoring was a much more severe problem in factor analysis than overfactoring.

Parallel Analysis



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Regression-Based Methods

- Cattell-Nelson-Gorsuch (CNG)
  - k = > 3, p = > 6
  - Uses 6 eigenvalues concurrently
  - Compares slopes of all possible sets of three adjacent eigenvalues
  - Greatest absolute value of difference between two successive slopes

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Regression-Based Methods (cont'd)

- Multiple Regression (MR)
  - k = > 3, p = > 6
  - Based on same principle as CNG
  - Utilizes all the eigenvalues in each comparison of the slopes
  - Greatest absolute value of difference between the slopes

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Regression-Based Methods (cont'd)

- t-value index (t)
  - k = > 3, p = > 6
  - A variation of the MR procedure
  - Slopes of the regression lines are compared using the usual formula for the t-test
  - Greatest absolute value of t

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Regression-Based Methods (cont'd)

- Standard Error of Scree (Se_{scree})
  - Errors of estimate are calculated using a sequence of regression analyses employing a decreasing number of eigenvalues.
    - $SE_{Y\,(e1\text{-}ev)}$ 1,2,3, ..., v
    - $SE_{Y\,(e2\text{-}ev)}$ 2,3,4, ..., v
    - $SE_{Y\,(e3\text{-}ev)}$ 3,4,5, ..., v
  - If SE > 1 / v

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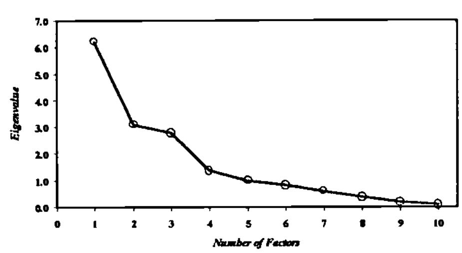
Slope Comparisons



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Residual Matrix Analysis

- Minimum Average Partial (MAP)
  - Based on matrices of partial correlations
  - After each of the factors is partialed out, the average of the squared partial correlation is calculated
  - Continued until the residual matrix most closely resembles an identity matrix

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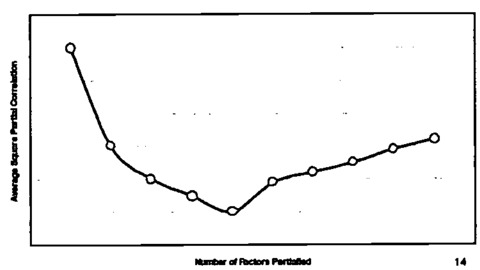
MAP Procedure



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Method

- Number of factors (3, 5, 7)
- Number of variables (*p*) in the correlation matrices (3*k*, 5*k*, 10*k*)
- Sample Size (3*p*, 5*p*, 10*p*, 20*p*, 40*p*)
- Level of Communality (low, wide, high)
- Interfactor Correlation (0, .2, .5, mixed)

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Method (continued)

- For each condition (i.e., interfactor correlations, communality level, *k*, *p*) we generated a random sample of 10 population R matrices
- From each population R matrix generated 1,000 samples of each size (N per *p*)

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Example of Population Pattern Matrix

(k=3 & p=15)

High			Wide			Low		
.87	-.01	.21	.89	-.03	-.03	.63	-.01	.07
.87	-.05	.19	.83	.04	-.04	.65	-.02	-.03
.86	.18	.18	.74	.49	.13	.45	-.02	-.02
.76	.47	.06	.69	.57	-.05	.44	-.01	.09
.77	-.03	-.03	.45	-.01	.04	.38	.23	.03
.61	-.02	.58	.18	.87	.08	.36	.24	.34
.69	.51	-.06	.16	.63	-.01	.11	.62	-.03
-.01	.89	.08	.15	.69	-.02	-.02	.55	-.02
-.04	.84	-.03	-.01	.66	.3	-.02	.46	-.02
.16	.82	-.04	-.03	-.04	.84	.24	.38	-.03
-.04	.78	.30	-.03	.11	.70	.21	-.03	.60
.40	.64	.36	.48	-.06	.68	.04	.00	.56
-.06	.10	.89	-.04	.59	.67	.10	-.01	.84
-.03	.03	.77	.18	-.03	.62	.15	.03	.63
.48	-.01	.69	.16	-.01	.42	.23	.14	.48

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Accuracy Criteria

- Performance was evaluated based on:
  - Average number of factors retained by each method
  - Proportion of samples retaining the same number of factors as the population
  - Proportion of samples retaining the same number of factors as the rule of thumb applied to the population

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Influence of Design Factors

- Considerable variability was evidenced across the conditions examined for each method.
- To investigate which design factors were associated with the most variability, $\hat{\omega}^2$ was computed and examined.

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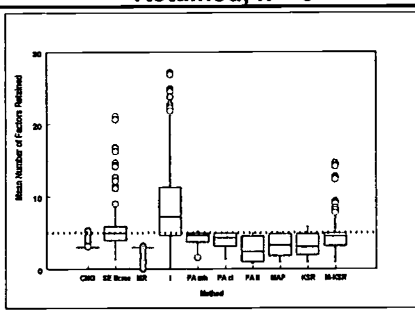
Distributions of Mean Number of Factors Retained, k = 5



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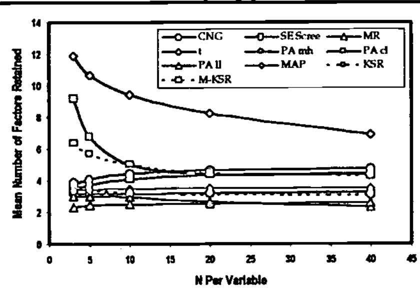
Mean Number of Factors Retained by N per Variable (N:p) for k = 5



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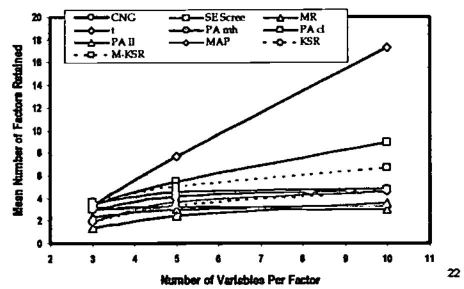
Mean Number of Factors Retained by Number of Variables Per Factor (p:k) for k = 5



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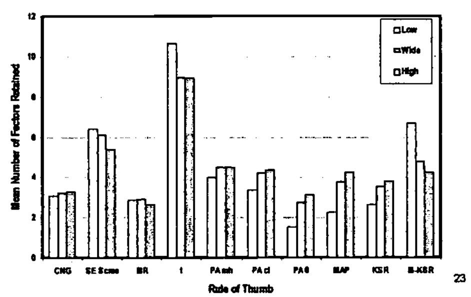
Mean Number of Factors Retained by Communality Type for k = 5



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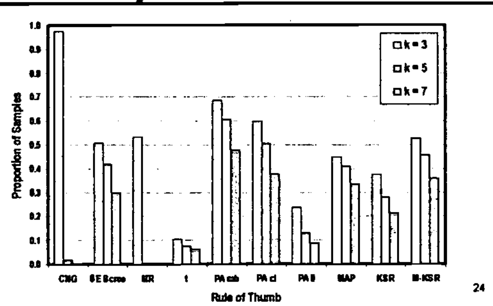
Proportion of Samples Retaining K Factors by True Number of Factors



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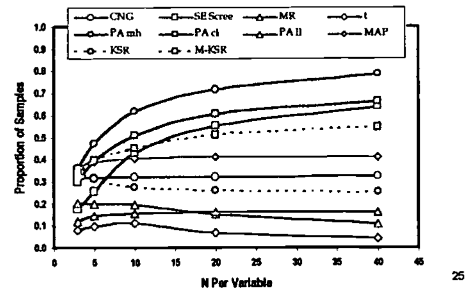
Proportion of Samples Retaining K Factors by N per Variable (N:p)



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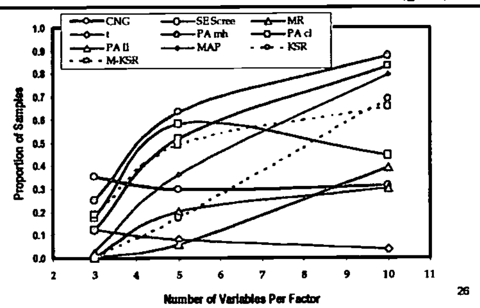
Proportion of Samples Retaining K Factors by Number of Variables Per Factor (p:k)



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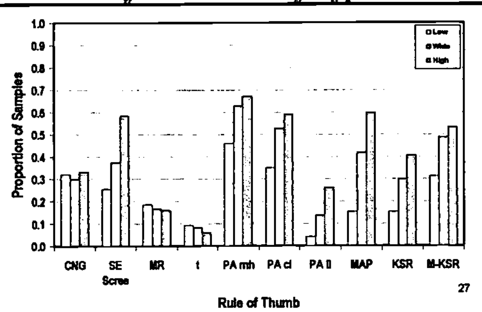
Proportion of Samples Retaining K Factors by Communality Type



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Results

- More variables per factor, larger samples, higher communality
- Most rules underestimated true number of factors
- PA_{MH} is best overall, but MAP is slightly better with small samples ($N = 3p$)
- Some methods rarely lead to correct number of factors

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Conclusions

- No method should be used in isolation
- The 'art of factor analysis'
- Some methods should be avoided
- What to do about low communality and small samples? Need a new and better thumb

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MacCallum et al. (2001) state "given the serious consequences of underfactoring, ...,users of factor analysis should be very rigorous in making the number-of-factors decision and should err in the direction of overfactoring when the evidence is ambiguous". Wood, et al. (1996), however, caution about the importance of extracting the correct number of factors, as they found in their studies that the factor solution with the least error is the one that extracts the correct number of factors.

Methods for Determining Number of Factors

The factor analysis literature is replete with recommendations regarding the most appropriate procedures to employ as well as an array of criteria to consider when addressing the number-of-factors problem (e.g., Gorusch, 1983; Hakstian, Rogers, & Cattell, 1982; Zwick & Velicer, 1986). According to Horn and Engstrom (1979), after 50 years of work in the field, no fewer than 50 tests had been invented. There is, however, little consistency in the results obtained from applications of these tests across data sets and varying conditions. Thus, many applied researchers are often in doubt about the efficiency of the procedures they use in seeking to determine reasonable estimates of the number of factors that underlie a given data structure.

Nasser, Benson, and Wisenbaker (2002) note that although researchers addressing the number-of-factors to retain problem have advanced numerous approaches based on different theoretical rationales (algebraic, statistical, psychometric, substantive importance, and interpretability), the most commonly employed methods for determining the number of factors in applied research are Kaiser's eigenvalue-greater-than-one rule (Kaiser, 1960) and Cattell's visual scree test (Cattell, 1966). Both methods are readily available in commonly used statistical software packages, which may account for their widespread use. However, several researchers have discussed the limitations of these two procedures and have warned against their use in making decisions regarding the number of factors to retain.

In the case of the Kaiser-greater-than-one rule (Kaiser, 1960), factor retention is based upon the number of eigenvalues greater than one. More specifically, the rule

calls for as many components or factors to be retained as there are eigenvalues greater than 1.0. This procedure was initially suggested by Guttman (1954) based on the consideration that it provided a lower bound for the number of common factors that underlie a correlation matrix having unities in the main diagonal. Turner (1998) argues that the Kaiser rule is flawed as it is based on an assumption that a factor is not psychometrically sound if it accounts for less variance than a single variable does. Cliff (1988) suggests that the rule lacks a logical basis as one of the rationales for the rule was based on a misapplication of a formula for estimating the reliability of a multi-item test. A number of researchers (e.g., Cattell & Jaspers, 1967; Fava & Velicer, 1992; Hakstian, Rogers, & Cattell, 1982; Lee & Comrey, 1979; Zwick & Velicer, 1986) have reported the use of the Kaiser rule as problematic as it often leads to an overestimate of the number of components or factors that underlie the data, thus giving rise to the potential problems associated with overfactoring. Others report a tendency to either overestimate or underestimate the number of components or factors depending on the conditions present in the data, including whether it is applied to population versus sample matrices (see for example, Cliff, 1988; Hakstian, Rogers & Cattell, 1982).

Cattell's scree test involves identifying the number of factors or components to retain based on a visual examination of the graph of eigenvalues plotted on the vertical axis and the factor sequence numbers plotted on the horizontal axis. The process involves separation of the "scree" of trivial factors from the "cliff" of nontrivial factors (Cattell, 1966). A straight edge is placed along the bottom part of the graph where the points form an approximately straight line (Cattell & Vogelman, 1977). The points above and to the left of the straight line correspond to the nontrivial factors; points on or close to the straight line constitute the trivial factors. This process of identifying the nontrivial factors and hence, the number of factors to retain, introduces a considerable amount of subjectivity on the part of the applied researcher. Several researchers have noted that issues of reliability often arise in the use of this procedure (e.g., Crawford & Koopman, 1979; Zwick & Velicer, 1986). The absence of an obvious break or the presence of multiple breaks in the eigenvalue pattern may make it difficult for one to

make a decision about the appropriate number of factors to retain (Jurs, Zoski, & Mueller, 1993; Nasser, Benson & Wisenbaker, 2002). Some researchers have reported reasonable accuracy of the scree test, however, the range of conditions examined in their studies were limited (see for example, Tucker, Koopman & Linn, 1969; Cattell & Jaspers, 1967; Zwick & Velicer, 1986). Zwick and Velicer (1986) indicated in their study of the number of components to retain that the scree test "was generally accurate but variable" and recommended against its use as the sole decision method.

In light of the subjective nature of Cattell's visual scree test, researchers have sought to develop more objective variations of the scree test in order to overcome some of its limitations. Four regression-based variations have been proposed, namely, the Cattell-Nelson-Gorsuch (CNG) test (Gorsuch & Nelson, 1981); and three procedures proposed by Zoski and Jurs (1993, 1996): a multiple regression approach (MR), a t-value index, and the standard error of scree (SE_{scree}). A recent Monte Carlo study conducted by Nasser, Benson, and Wisenbaker (2002) compared the performance of these four regression-based methods under a variety of simulated conditions. The researchers reported the SE_{scree} procedure to be the most accurate in terms of its performance with both correlated and uncorrelated factors. The other three methods were found to consistently underestimate or overestimate the number of factors to be retained. These researchers noted several limitations of their study, including the range of conditions examined. They suggested that this investigation should be viewed as an initial but crucial phase in the investigation of the efficiency of regression-based procedures for determining the number of factors, as previous work in the field was based solely on a few correlation matrices from the published literature.

Additionally, other methods considered to be more effective than Kaiser's rule and Cattell's scree test in estimating the number of factors that underlie a data set have been proposed. These include parallel analysis (Horn, 1965; Montanelli & Humphreys, 1976; Lautenschlager, Lance, & Flaherty, 1989) and a minimum-average partial (MAP) correlation procedure proposed by Velicer (1976).

Given the need for a more comprehensive study and comparison of the effectiveness of methods proposed for determining the number of factors to retain in exploratory factor analysis, we undertook an investigation of ten methods that can be classified under four broad categories of procedures: the Kaiser criterion; parallel analysis; regression-based variations of the scree test, and residual matrix analysis. The ten methods examined are briefly described below.

Description of Methods

Kaiser Rule

Eigenvalue Greater Than 1.0 (KSR). This is perhaps the most commonly used method for determining the number of components to retain in principal component analysis and is also frequently employed by more than novice researchers in the conduct of exploratory common factor analysis. Zwick and Velicer (1986) suggest that the popularity of its use may be due to its availability as the default option in many statistical packages. The rationale for this method was initially proposed by Guttman (1954) and later adapted and popularized by Kaiser (1960). Factors or components with eigenvalues greater than 1.0 (i.e., components which evidence at least as much variance as one of the original variables) are retained in the analysis and subjected to rotation. As noted earlier, several researchers have indicated that Kaiser's eigenvalue-greater-than-1 rule is problematic (e.g., Yeomans, & Golden, 1982; Zwick & Velicer, 1986; Wood, Tataryn, & Gorusch, 1996) and have advised against its use in factor analysis. We include it here because of its common use, and our desire to compare this method with more recently developed approaches.

Eigenvalue Greater Than Average Eigenvalue in Sample (M-KSR). This modification of the Kaiser rule was first proposed by Guttman (1954). This method, which may be more appropriate than the original KSR for common factor analysis, calls for the retention of factors with eigenvalues greater than the average eigenvalue in the sample.

Parallel Analysis

This method was developed by Horn (1965). It involves the comparison of eigenvalues of a correlation matrix of p random uncorrelated variables with those of the eigenvalues of the correlation matrix of p variables in the actual data set, based on the same sample size. Factors of the real data set that have eigenvalues greater than the eigenvalue of the corresponding factor in the random data set are retained and subjected to rotation.

Parallel Analysis (PA_{MH}). A regression equation for predicting the eigenvalues of random correlation matrices with squared multiple correlations on the main diagonal of the random correlation matrix was subsequently developed by Montanelli and Humphreys (1976):

$$\ln(\lambda_i) = a_i + b_i \ln(N-1) + c_i \ln\{(.5)(k(k-1)) - (i-1)k\}$$

where N = number of observations,

k = number of variables in the correlation matrix, and

i = eigenvalue sequence number.

The regression weights (a_i, b_i, c_i) were obtained via simulation methods and are provided by Montanelli and Humphreys (1976).

A decision rule was suggested based on the point at which the plot of sample eigenvalues crosses the plot of expected eigenvalues from matrices of random data (Montanelli & Humphreys, 1976). Factors above the point at which the two lines cross, were retained for rotation. Factors that lie below the point that the plots crossed (i.e., those factors whose eigenvalues were less than the eigenvalues of the corresponding factors of the random data set) were considered trivial factors and were not retained.

Humphreys and Montanelli (1975) found the parallel analysis method to provide accurate estimates of the number of factors to be retained in common factor analysis. Zwick and Velicer (1986) also reported that within the context of principal components analysis, the parallel analysis method yielded consistently accurate estimates of the number of components to retain. In fact, they indicated that in their study in which they compared five different rules for determining the number of components to retain,

the parallel analysis method "was typically the most accurate method at each level of complexity examined."

Parallel Analysis Close (PA_{CL}). A modification of Horn's parallel analysis procedure is to eliminate factors with eigenvalues that are close in magnitude but not necessarily less than the magnitude of the corresponding eigenvalues of the random data. Such a modified criterion is expected to retain fewer factors than the original Horn procedure. For this study, sample eigenvalues within .10 of the random data eigenvalues were considered close enough to stop retaining factors.

Modified Parallel Analysis (PA_{LL}). Lautenschlager, Lance and Flaherty (1989) proposed a modification to the general regression equation first developed by Montanelli and Humphreys (1976) and further refined by Allen and Hubbard (1986) to predict eigenvalues of factors resulting from a random data correlation matrix. These researchers noted that the Allen and Hubbard's equation did not yield an accurate estimate of the first eigenvalue; this in turn directly influenced the accuracy of subsequent eigenvalues in the series. They modified the equation proposed by Allen and Hubbard (1986) by adding a variables-to-subjects ratio term:

$$\ln(\lambda_i) = a_i + b_i \ln(N-1) + c_i \ln\{(.5)(k-i-1)(k-i+2)\} + d_i \ln(\lambda_{i-1}) + e_i\,(k/N)$$

The regression weights (a_i, b_i, c_i, d_i and e_i) are provided by Lautenschlager, Lance and Flaherty (1989).

The results of a Monte Carlo study using the newly augmented regression equation showed that it served to significantly improve the prediction of the first, and consequently subsequent eigenvalues (up to 48) of a random data matrix (Lautenschlager, et al., 1989) when compared to eigenvalues generated from the Allen and Hubbard (1986) equation. These researchers recommend use of the modified regression equation with the added variables-to-subject ratio term in future parallel analysis applications.

Regression-Based Variations of the Visual Scree

Each of the methods described in this category is based on Cattell's (1978) guidelines for determining the number of common factors to retain.

Cattell-Nelson-Gorsuch (CNG). In an attempt to find a more objective approach to the visual scree test, Gorsuch and Nelson (1983) developed an analytical method using multiple linear regression to determine the number of factors to retain. In this procedure, the slopes of all possible sets of three adjacent eigenvalues each are compared. More specifically, the slope of the first three points (roots) in the eigenvalue plot (1, 2, and 3) is compared with the slope of the next three points (4, 5,and 6). Then the slope of points 2, 3, and 4 is compared with the slope of points 5, 6,and 7, and so on until all points have been incorporated in the comparisons. The number of factors to be retained is defined by the point at which the difference between the two successive slopes is greatest.

Multiple Regression (MR). The CNG procedure uses only six eigenvalues (i.e., six data points) at a time to determine each pair of slopes for comparison. Zoski and Jurs (1993) suggest that this procedure uses only a limited amount of information to determine the number of factors to retain. They therefore proposed a multiple linear regression approach that would include more data points in computing the slope of the regression lines. The MR approach is based on the same principle as the CNG, but it utilizes all the eigenvalues in each comparison of pairs of slopes. A series of adjacent slopes are obtained and all possible successive pairs of slopes are compared. One slope in each paired comparison is based on an increasing number of eigenvalues and corresponds with the major factors; the second slope is based on a decreasing number of eigenvalues and corresponds with the trivial (scree) factors. More specifically, the slope of the regression line developed from points 1, 2, and 3 in the eigenvalue plot is compared to the slope of the regression line developed from points 4, 5, 6, ..., p (where p is the number of eigenvalues in the plot). Then, the slope of the regression line for points 1, 2, 3, and 4 is compared to the slope for points 5, 6, 7, ..., p, and so on. This process continues until all adjacent slopes of all possible successive pairs of regression lines are compared. The decision regarding the number of factors to retain corresponds

to the point at which the absolute value of the difference between the slopes is the greatest.

t-value index (t). Another procedure examined introduced a variation of the MR procedure known as the t-value index. Using the slopes obtained in the MR procedure, the slopes of the regression lines are compared using the usual formula for the t-test of the difference between slopes. The number of factors to retain is based on the largest absolute value of t.

Standard Error of Scree (SE_{scree}). Because the CNG, MR, and t-value index are not applicable when the number of factors is less than three and/or the number of variables is less than six, Zoski and Jurs (1996) developed the standard error of scree (SE_{scree}) procedure which is based on the standard error of estimate for a set of points in the plot of the eigenvalues. In this procedure, the errors of estimate are calculated using a sequence of regression analyses employing a decreasing number of eigenvalues. Initially, all eigenvalues are regressed onto their ordinal numbers. The procedure continues with subsequent sets of eigenvalues and concludes when the standard error of estimate meets the $1/p$ criterion. Because the error variance tends to be inversely related to sample size, Zoski and Jurs (1996) set the value of $1/p$ as the criteria for determining the number of factors to retain (i.e., the number of standard errors that exceed $1/p$ is the number of factors to retain). Zoski and Jurs (1996) and Nasser, et al. (1996) found SE_{scree} to be more accurate in identifying the number of factors to retain than the CNG and multiple regression (MR) procedures.

Residual Matrix Analysis

Minimum Average Partial (MAP) Correlation. The Minimum Average Partial (MAP) method developed by Velicer (1976) is based on a matrix of partial correlations. After each of the factors has been partialed out, the average of the squared partial correlations is calculated. When the residual matrix most closely resembles an identity matrix, no further factors are extracted and rotated. Using this method, at least two variables will have high loadings on each retained factor. Zwick and Velicer (1986)

reported the MAP to be one of the two most accurate methods (the other being parallel analysis) in determining the number of components to retain in their study of five decision rules in principal components analysis.

Purpose

The purpose of this study was to compare the effectiveness of the ten aforementioned methods (the Kaiser rule (KSR), a modified Kaiser criterion (M-KSR), three variations of parallel analysis-- Montanelli and Humphrey's (PA_{MH}), parallel analysis--close (PA_{CL}), and Lautenschlager et al. modified parallel analysis (PA_{LL}), four regression-based variations of the scree procedure (CNG, MR, t, SE_{scree}), and the minimum-average partial (MAP) procedure) in determining the number of factors to retain in exploratory common factor analysis. The performance of these procedures was evaluated based on the average number of factors retained by each method, the proportion of samples retaining the same number of factors as the true number of factors in the population, and the proportion of samples retaining the same number of factors when a particular rule of thumb is applied to the population.

Method

The performance of the ten procedures were investigated using Monte Carlo methods, in which random samples were generated under known and controlled population conditions. The population correlation matrices varied with respect to the particular aspects of interest. Population correlation matrices were constructed based on the methods described by Tucker, Koopman and Linn (1969), which have been used in large-scale simulation studies (MacCallum, et al., 1999; Tucker et al., 1969). For correlated factors, a generalization of the method was utilized that was described by Hong (1999). The true factor loading patterns underlying these matrices were constructed to exhibit relatively clear simple structure. For these population correlation matrices, the number of measured variables, the number of common factors, the level of communality and the correlation between factors were controlled. Sample

correlation matrices were generated from each of the known population correlation matrices. Each of the sample matrices was then analyzed using principal axis factor extraction. The pattern of eigenvalue magnitude was used to determine the number of factors that should be retained.

Generation of Population Matrices

In the Monte Carlo study, uncorrelated population correlation matrices were generated using the method presented by Tucker, Koopman, and Linn (1969). These matrices were generated under the assumption that the common factor model holds exactly in the population. This method produces population matrices leading to relatively clear simple structure and has been used in other simulation studies (MacCallum et al. 1999; Mundfrom, Shaw & Ke, 2001; Tucker, Koopman and Linn, 1969). The population R is generated based on major, minor, and unique factors,

$$R = A_1A_1' + A_2A_2' + A_3A_3'$$

where A_1 is the *pxk* matrix of actual input factor loadings for the major factors, A_2 is the matrix of actual input factor loadings for the minor factors, and A_3 is the *pxp* diagonal matrix of actual input factor loadings for the unique factors. The contribution of the minor factors (A_2) was set to zero in this study so that the data generation model matched a factor analytic model with *k* common factors. The common factors were specified as orthogonal.

The process for creating A_1 starts with the creation of a matrix of conceptual input factor loadings, $\widetilde{A}_1$. To create $\widetilde{A}_1$, the loading of a variable on a randomly selected factor, *j*=1 thru *k*, is set to a value randomly chosen between 0 and *k*-1, (for a 3 factor model $\widetilde{a}_{1j}$ could be 0, 1, or 2). Next the loading on a randomly selected factor from those remaining is set to a value randomly chosen between 0 and *k*-1-$\widetilde{a}_{1j}$. This process continues until a conceptual input factor loading has been chosen for each factor, and ensures the sum of the loadings across the factors is *k*-1. This process is then repeated for each of the *p* variables.

The matrix of actual input factor loadings, A_1, is then created from the matrix of conceptual input factor loadings, $\tilde{A}_1$, through a series of three steps: (1) normal deviates are added to introduce error, (2) a skewing function is used to limit negative factor loadings, and (3) the matrix is scaled to ensure desired levels of communality. The diagonal matrix, A_3, for the unique factors is also scaled to ensure the desired levels of communality. The levels of communality (h^2) that were simulated were high (h^2 for each variable drawn randomly from values of .6, .7, and .8), wide (h^2 for each variable drawn randomly from values of .2, .3, .4, .5, .6, .7, and .8), or low (h^2 for each variable drawn randomly from values of .2, .3, and .4), which are consistent with the levels used in other simulation studies. An example matrix of input factor loadings, A_1, that was created using these methods is presented in Table 1 (with k=3 and p=15) for each level of communality.

To construct population matrices for correlated factors, the above method was modified following Hong (1999). More specifically,

$$R = JBJ' + A_3A_3'$$

where

$$J = [A_1 \ A_2]$$

and

$$B = \begin{bmatrix} \Phi & \Upsilon \\ \Upsilon' & \Gamma \end{bmatrix},$$

where Φ is a matrix of correlations among major factors, Γ is a matrix of correlations among minor factors, and Υ is a matrix of correlations among major and minor factors. The simulations conducted for this study were somewhat simplified since the contributions of minor factors (A_2, Γ, Υ) were set to zero.

It should be noted that this method of generating population correlation matrices controls the number of measured variables, the number of common factors, and the level of communality, but that more than one population correlation matrix can be generated having the desired number of items, factors, and communality. The

specifications define a general class of population matrices, but the random selection involved in the creation of the conceptual input loading matrix, the introduction of error, and the random selection of specific communality values all influence the population correlation matrix that is obtained.

Monte Carlo Study Design

The Monte Carlo study included five factors in the design. These factors were (a) the number of common factors, *k*, present in the population (populations were simulated with $k = 3$, 5 and 7 factors), (b) the number of variables, *p*, in the correlation matrices (with $p = 3*k$, $5*k$ and $10*k$), (c) the level of communality, (high, wide, and low), (d) the level of interfactor correlation (with $r_{ij} = 0$, .3, .5 and a mixed condition with interfactor correlations ranging from 0 to .5), and (e) sample size, *N* (with samples of 3*p*, 5*p*, 10*p*, 20*p*, and 40*p*). These *N:p* ratios represent values that range from those generally considered insufficient to those considered more than adequate (MacCallum et al., 1999). The factors in the design were crossed with each other, providing a total of 540 conditions in the Monte Carlo study.

The research was conducted using SAS/IML version 8.2. Conditions for the study were run under Windows 98 and Windows 2000. For each condition investigated, 10 population matrices were generated. Then for each population matrix, 1,000 samples were generated. This provided the opportunity to examine the degree to which the results varied across different population matrices within a condition. The use of 10,000 samples provided adequate precision of estimates of the sampling behavior of the factor recovery indices. For example, 10,000 samples provide a maximum 95% confidence interval width around an observed proportion that is ± .0098 (Robey & Barcikowski, 1992).

Results

Mean Number of Factors Retained

The mean number of factors obtained by each method is presented for all combinations of *k*, *p*, *N*, and communality in Tables 2-5, where each table contains a different level of correlation among the factors. A quick perusal of these tables suggests that the mean number of factors being retained differs across methods, and for each particular method the closeness between the mean number of factors retained and the true number of factors, *k*, varies across the conditions studied, with larger sample sizes and more variables tending to lead to better results.

To summarize the performance of the methods, a series of box plots was created. Each box plot shows the distribution of the mean number of factors retained for a particular method. These box plots are presented in Figures 1-3, for $k = 3, 5$, and 7 factors, respectively. For the three-factor condition (Figure 1), the CNG method has a mean number of factors that is very close to 3, the true value, for all conditions. The other methods show somewhat more variability across conditions. Interestingly, if one examines the five-factor conditions (Figure 2), the CNG is still tending to suggest 3 factors, and it continues to suggest three factors even when the true number of factors is seven (Figure 3). Thus its effectiveness drops off considerably as the number of factors increases.

Two of the parallel analysis methods (PA_{MH} and PA_{CL}) appear to do relatively well in the sense that the mean number of factors retained is close to the true number of factors for a relatively large portion of the conditions. Note that when these methods produce means that differ from *k*, the means tend to be smaller, suggesting too few factors are being retained. The tendency to have too few factors is also found and somewhat more prevalent for the PA_{LL}, MAP, and KSR methods. The distributions for t, SE_{scree}, and M-KSR show the most variability, and are the only methods that tend to have an average number of retained factors that exceeds *k* for many conditions.

Since the mean number of factors retained differs across conditions for each method, $\hat{\omega}^2$ was used to determine which design factors were associated with the most

variability. These values are presented in Table 6. Note that one would hope that the majority of the variation in the mean number of factors retained would be associated with k, the true number of factors. The only method for which over 50% of the variation was attributable to *k* was the PA_{CL} method ($\hat{\omega}^2 = .61$). Figure 4 contains a graph showing the mean number of factors retained for each method as a function of *k*. As *k* increases from 3 to 5 to 7 the mean number of factors retained also increases for each method except the CNG, which stays level at 3 factors extracted. Ideally the bars would reach, but not exceed, values of 3, 5, and 7, and the method that is coming closest to this is the M-KSR.

The $\hat{\omega}^2$ analysis also shows that the mean number of factors retained varies with other design factors. Figure 5 shows variation as a function of sample size per variable (*N/p*) for each method. This figure displays results for conditions with $k = 5$, but the pattern was consistent across the values of *k* investigated. With increasing *N/p*, the PA_{MH}, PA_{CL}, and M-KSR tend to get close to the expected value of 5. With small *N/p* ratios M-KSR tends to retain too many factors, while PA_{MH} and PA_{CL} tend to retain too few factors.

The effect of the number of variables per factor (*p/k*) is shown in Figure 6. The methods tend to have a larger mean number of retained factors when the number of variables per factor is greater. When the number of variables per factor is relatively small (3), all methods tend to underestimate the number of factors. When number of variables per factor is large (10), four methods have a mean number of retained factors that is near the true value of 5. These are the PA_{MH}, PA_{CL}, MAP and KSR methods.

The effects of communality are depicted in Figure 7. As communality increases the mean number of factors retained by the methods tends to get closer to the true value of 5. For some methods this implies the mean number of retained factors decreases with communality while for others it increases with communality. Phi also appears to have some effect on the mean number of factors retained for most methods (Figure 8). Generally the mean number of retained factors decreases with increases in the correlations among factors.

Proportion of Samples Retaining the Same Number of Factors as the Population

In addition to the mean number of factors obtained, the extent to which the number of factors retained in each sample matrix matched the number of factors present in the population correlation matrix was examined. The proportion of agreement between the sample matrices and the population matrices (i.e., the proportion of samples retaining k factors) is presented for all combinations of k, p, N, and communality in Tables 7 – 10, aggregated among each level of inter-factor correlation. A review of these tables reveals that the proportion of samples in agreement with the population differs considerably across the methods and conditions investigated. Generally, larger sample sizes, a greater number of variables per factor, high communality levels, and low inter-factor correlations demonstrated better results.

Due to evidence of large variability across the conditions examined for each method, $\hat{\omega}^2$ was computed to determine which design factors were associated with the most variability in the proportions. Table 11 displays these results. Contrary to the expectations about $\hat{\omega}^2$ in relation to the number of factors extracted, it was expected that little variation in the proportion of agreement would be associated with k, the true number of factors underlying the population correlation matrix. One method for which the majority of variation was attributable to k was the CNG method ($\hat{\omega}^2$=.98). In addition, a considerable amount of variation that was attributable to k was the MR procedure ($\hat{\omega}^2$=.48). As demonstrated in Figure 9, as k increased, the proportion of samples in agreement with the population decreased for all of the methods. Although the CNG method demonstrated excellent results for k = 3 matrices, agreement dropped dramatically as the number of factors increased. Additionally, for the MR procedure, agreement dropped to zero for populations of k = 5 and k = 7.

The $\hat{\omega}^2$ analysis also revealed that the proportion of samples in agreement with the population varied with other design factors. Figure 10 depicts the variation in proportions as a function of sample size per variable (N/p) for each method. With increasing N/p, the majority of the methods evidenced an increase in agreement, with

the PA_{MH} method exhibiting superior performance. In addition, the SE_{scree} method evidenced the greatest improvement in factor retention with increasing N/p.

The influence of the number of variables per factor (p/k) on the proportion of samples in agreement with the population is shown in Figure 11. For the majority of the methods, the proportion of samples in agreement with the population increased as the number of variables per factor increased, with the PA_{MH} and PA_{CL} methods demonstrating exceptional performance. In particular, the MAP procedure showed the most dramatic improvements in agreement as the number of variables per factor increased. In contrast, the CNG and t procedures demonstrated the opposite trend, declining in their agreement with the population as the number of variables per factor increased. Interestingly, the SE_{scree} method showed significant improvement in the level of agreement as the number of variables per factor increased until the p/k ratio was 5. At that point, the proportion of samples in agreement with the population declined.

The effects of communality on the proportion of samples in agreement with the population were also examined. As shown in Figure 12, the majority of the methods evidenced that as communality increased, the proportion of samples in agreement with the population increased. In contrast, the MR and t methods evidenced a decrease in performance with an increase in communality level. Phi also appeared to affect the proportion of agreement for almost all methods (Figure 13). With the exception of the CNG and MR methods, agreement decreased as the interfactor correlation increased. Negligible differences were observed for the CNG and MR procedures as the correlation among factors increased.

Proportion of Samples Retaining the Same Number of Factors as the Rule of Thumb Applied to the Population

Lastly, an examination was made of the extent to which the number of factors retained by each rule matched the number of factors that would have been retained if we had used the rule on the population correlation matrix. The agreement with that which we would have seen in the population is presented for all combinations of k, p, N,

and communality in Tables 12-15, with each table displaying a different level of inter-factor correlation. An examination of these tables suggests that the proportion of samples differs considerably across the methods, and for each method this proportion is seen to vary across the conditions studied. In general, conditions with larger sample sizes and more variables per factor evidenced better results.

Because the proportions evidenced variability across the conditions examined for each method, we computed a series of $\hat{\omega}^2$ values to investigate which design factors were associated with the most variability in these proportions. These values are presented in Table 16. Consistent with the results presented above, with regard to agreement between the sample matrices and the population matrices, we would not expect a sizeable portion of the variation in agreement to be associated with *k*, the true number of factors underlying the population correlation matrix. The only method for which a considerable amount of variation was attributable to *k* was the MR method ($\hat{\omega}^2 = .46$). Figure 14 illustrates the proportion of samples agreeing with the rule applied to the population R-Matrix as a function of *k*. As *k* increases, the proportion of samples in agreement with the population decreased for most of the methods with the exception of MR, which decreased with $k = 5$ but increased substantially with $k = 7$. Additionally, the CNG remained zero regardless of the number of factors. Finally, for the t procedure, agreement dropped quite dramatically as the number of factors increased.

The $\hat{\omega}^2$ analysis revealed that the proportion of samples in agreement with the population also varied with other design factors. Figure 15 shows variation in proportions as a function of sample size per variable (*N/p*) for each method. With increasing *N/p*, the majority of the methods evidenced an increase in agreement, with the non-scree based methods exhibiting superior performance. However, the SE_{scree} method evidenced the biggest improvement in factor retention with increasing *N/p*.

The influence of the number of variables per factor (*p/k*) is shown in Figure 16. It was our expectation that a considerable influence would be associated with p/k. Three methods were observed for which a notable amount of variation was attributable to p/k; t, PA_{MH} and PA_{CL} ($\hat{\omega}^2 = .28$, $\hat{\omega}^2 = .28$ and $\hat{\omega}^2 = .24$ respectively). The non-scree

based methods tended to fare slightly better or remain constant as the number of variables per factor increased. The opposite trend was evidenced for the scree-based methods, all of which decreased in agreement as the number of variables per factor is increased.

Finally, the effects of communality are depicted in Figure 17. As communality increases the proportion of samples agreeing with the population tended to increase for the majority of methods examined, with the exception of the PA_{LL} method, which evidenced a decrease in performance with increased communality. Phi also appeared to have some effect on the proportion of agreement for most methods (Figure 18). Generally, agreement decreased as the interfactor correlation increased for the parallel analysis approaches (PA_{MH}, PA_{CL} and PA_{LL}) and the methods based on the Kaiser criterion (KSR and M-KSR). Negligible differences were observed for the MAP and the t procedures as the correlation among factors increased, however, the MAP procedure outperformed all other methods for the conditions examined.

Discussion

This empirical investigation of methods to determine the number of factors to retain in common factor analysis clearly suggests that both the choice of method and the design of the factor analytic study play crucial roles in retaining the correct number of common factors. For example, with phenomena characterized by low communality of variables, a study designed with few variables per factor and small sample size is unlikely to lead to the correct number of factors, regardless of the method selected (except for the trivial case of the success of the CNG method when the true number of factors is 3). Investigations in such areas should include larger numbers of variables (i.e., $p = 10k$) and a very large number of observations ($N = 40p$). With such a design, the SE_{scree}, parallel analysis (both PA_{MH} and PA_{CL}) and the M-KSR methods provided nearly 100% accuracy across levels of k and inter-factor correlation. Phenomena that are characterized by higher levels of communality appear to have less stringent data requirements, especially if the correlation between factors is low.

The most prevalent type of error made in the number-of-factors problem appears to be one of underfactoring if the parallel analysis methods (PA_{MH}, PA_{CL}, PA_{LL}), the MAP or the Kaiser methods (KSR, M-KSR) methods are used. Turner (1998) also found that under some circumstances, parallel analysis may underestimate the number of factors that are in the data. For the methods based on fitting regression lines to the scree plot, underfactoring was also characteristic of CNG and MR, while the t procedure showed a tendency towards overfactoring (especially as the true number of factors increased). The SE_{scree} method appeared to offer a balance between overfactoring and underfactoring, although it showed a tendency to estimate an exceptionally large number of factors in conditions with small sample sizes.

In terms of overall accuracy across the conditions examined in this study, the PA_{MH} approach to parallel analysis provided the largest proportion of samples retaining the correct value of k, an advantage seen across levels of communality, inter-factor correlation and values of k (except for $k = 3$, in which the CNG approach was the most accurate). Across sample sizes, the PA_{MH} method was the most accurate except for the smallest samples examined ($N = 3p$), in which the MAP evidenced a slight advantage. Similarly, for values of p/k, the PA_{MH} was the most accurate except for the smallest value ($p/k = 3$) in which the CNG provided the highest average accuracy. Interestingly, the more recent, modified prediction equations provided by Lautenschlager, Lance and Flaherty (1989) substantially reduced the accuracy in number-of-factors determination. These results, of course, must be considered in the light of the limitations of this research. Although the simulation approach we have followed has examined a range of values for p, k and N, and a variety of communality levels and levels of inter-factor correlation, further research is recommended to extend these findings. Additional work is also needed in the development of methods for accurately determining the number of factors in the more challenging conditions revealed here (e.g., low communality, few variables per factor and small sample sizes), conditions in which none of these rules of thumb were successful.

Such additional work is important because exploratory factor analysis is a frequently used multivariate technique in educational research. In the conduct of actual exploratory factor analyses, researchers face critical decisions regarding the number of factors to extract and retain. The interpretations gleaned from factor analytic solutions depend in large part upon the appropriate use of factor retention strategies. As exploratory factor analysis continues to enjoy a prominent position among the currently available multivariate methods, researchers must remain mindful of the limitations of certain procedures and methods. The results of this study underscore the need to exercise caution in factor retention decisions and highlight the need to consider, in the planning stages, those aspects of the factor solution that are important in the research application. This research furnishes valuable information about the sensitivity of commonly employed methods and provides guidance regarding the choice of alternative strategies.

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Table 1

Example Population Pattern Matrices as a Function of Communality when k=3 and p=15.

High Communality			Wide Communality			Low Communality		
.87	-.01	.21	**.89**	-.03	-.03	**.63**	-.01	.07
.87	-.05	.19	**.83**	.04	-.04	**.55**	-.02	-.03
.86	.18	.18	**.74**	.49	.13	**.45**	-.02	-.02
.76	.47	.06	**.69**	.57	-.05	**.44**	-.01	.09
.77	-.03	-.03	**.45**	-.01	.04	**.38**	.23	.03
.61	-.02	.58	.18	**.87**	.08	**.36**	.24	.34
.58	.51	-.06	.16	**.69**	-.01	.11	**.62**	-.03
-.01	**.89**	.08	.15	**.69**	-.02	-.02	**.55**	-.02
-.04	**.84**	-.03	-.01	**.56**	.3	-.02	**.45**	-.02
.16	**.82**	-.04	-.03	-.04	**.84**	.24	**.38**	-.03
-.04	**.78**	.30	-.03	.11	**.70**	.21	-.03	**.60**
.40	**.64**	.36	.48	-.06	**.68**	.04	.00	**.55**
-.06	.10	**.89**	-.04	.59	**.67**	.10	-.01	**.54**
-.03	.03	**.77**	.18	-.03	**.52**	.15	.03	**.53**
.48	-.01	**.69**	.16	-.01	**.42**	.23	.14	**.48**

Note. Largest loading for each variable is bolded.

Table 2
Mean Number of Factors Retained, Phi= 0.00

			Low Communality										Wide Communality										High Communality									
k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	3.3	2.3	0.0	0.0	2.1	1.6	0.5	1.1	1.7	3.3	3.1	2.5	0.0	0.0	2.3	2.3	1.0	1.5	1.9	1.9	3.1	2.6	0.0	0.0	2.4	2.3	2.4	2.0	2.2	2.4
		5p	3.2	1.8	0.0	0.0	2.2	1.8	0.5	1.1	1.3	3.1	3.1	2.3	0.0	0.0	2.5	2.5	1.1	1.5	1.7	1.7	3.1	2.6	0.0	0.0	2.5	2.5	2.5	2.1	2.1	2.4
		10p	3.1	1.7	0.0	0.0	2.3	1.9	0.6	1.1	1.1	2.8	3.0	2.3	0.0	0.0	2.7	2.7	1.2	1.5	1.5	1.5	3.1	2.6	0.0	0.0	2.6	2.6	2.6	2.2	2.1	2.4
		20p	3.1	1.7	0.0	0.0	2.4	2.0	0.7	1.0	1.1	2.5	3.0	2.4	0.0	0.0	2.8	2.8	1.2	1.4	1.4	1.4	3.1	2.6	0.0	0.0	2.7	2.6	2.7	2.3	2.1	2.4
		40p	3.1	1.8	0.0	0.0	2.4	2.1	0.8	1.0	1.0	2.3	3.0	2.4	0.0	0.0	2.9	2.9	1.3	1.4	1.4	1.4	3.1	2.6	0.0	0.0	2.7	2.6	2.7	2.4	2.1	2.4
3	15	3p	3.1	4.4	3.1	4.3	3.2	2.6	1.2	1.6	2.7	5.1	3.1	4.0	3.1	4.2	3.0	3.0	2.0	2.7	2.9	2.9	3.1	3.1	3.2	4.0	3.0	2.9	3.0	2.9	2.9	3.0
		5p	3.1	3.3	3.1	4.0	3.2	2.7	1.2	1.7	2.2	4.7	3.1	3.2	3.1	4.1	3.0	3.0	2.2	2.8	2.8	2.8	3.1	3.0	3.2	3.9	3.0	3.0	3.0	3.0	2.9	3.0
		10p	3.1	2.9	3.0	3.7	3.2	2.9	1.2	1.7	2.0	4.0	3.1	3.0	3.0	3.9	3.0	3.0	2.4	2.9	2.8	2.8	3.1	3.0	2.6	3.1	3.0	3.0	3.0	3.0	3.0	3.0
		20p	3.1	2.9	2.1	2.5	3.1	3.0	1.3	1.7	1.8	3.2	3.1	3.0	1.7	2.2	3.0	3.0	2.5	2.9	2.8	2.8	3.1	3.0	0.3	0.4	3.0	3.0	3.0	3.0	3.0	3.0
		40p	3.0	3.0	0.2	0.2	3.0	3.0	1.3	1.7	1.8	3.0	3.1	3.0	0.2	0.3	3.0	3.0	2.6	2.9	2.8	2.8	3.1	3.0	0.0	0.0	3.0	3.0	3.0	3.0	3.0	3.0
3	30	3p	3.2	10.2	3.2	4.8	3.2	3.0	2.8	3.0	3.6	8.8	3.1	8.1	3.1	4.4	3.0	3.0	3.0	3.0	3.0	3.0	3.1	4.7	3.1	4.0	3.0	3.0	3.0	3.0	3.0	3.0
		5p	3.2	6.6	3.3	4.1	3.1	3.0	2.9	3.0	3.0	7.6	3.1	5.7	3.1	4.1	3.0	3.0	3.0	3.0	3.0	3.0	3.1	3.2	3.2	4.0	3.0	3.0	3.0	3.0	3.0	3.0
		10p	3.2	3.6	3.3	4.1	3.0	3.0	2.9	3.0	3.0	5.5	3.1	3.5	3.2	4.1	3.0	3.0	3.0	3.0	3.0	3.0	3.1	3.0	3.2	4.1	3.0	3.0	3.0	3.0	3.0	3.0
		20p	3.2	3.0	3.4	4.1	3.0	3.0	3.0	3.0	3.0	3.4	3.1	3.0	3.2	4.0	3.0	3.0	3.0	3.0	3.0	3.0	3.1	3.0	3.2	4.1	3.0	3.0	3.0	3.0	3.0	3.0
		40p	3.2	3.0	3.4	4.1	3.0	3.0	3.0	3.0	3.0	3.0	3.1	3.0	3.2	4.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.2	4.1	3.0	3.0	3.0	3.0	3.0	3.0
5	15	3p	3.3	4.8	3.2	4.6	3.8	2.8	0.8	1.3	2.8	5.5	3.2	4.8	3.2	4.4	3.8	3.8	1.7	2.2	3.1	3.1	3.5	4.8	3.3	5.0	4.2	4.0	4.2	3.0	3.7	4.2
		5p	3.2	3.8	3.1	4.3	4.0	3.0	0.9	1.3	2.1	5.3	3.2	4.2	3.1	4.2	4.1	4.1	1.9	2.2	2.7	2.7	3.6	4.8	3.3	4.5	4.6	4.3	4.6	3.2	3.6	4.2
		10p	3.1	3.4	3.1	4.0	4.4	3.4	1.0	1.2	1.7	4.9	3.2	4.0	3.1	3.8	4.5	4.5	2.0	2.3	2.5	2.5	3.7	4.8	2.2	2.6	4.8	4.6	4.8	3.4	3.6	4.2
		20p	3.1	3.5	2.4	2.9	4.6	3.8	1.0	1.1	1.4	4.5	3.2	4.0	2.1	2.5	4.7	4.7	2.1	2.3	2.4	2.4	3.9	4.8	0.2	0.2	4.9	4.8	4.9	3.4	3.6	4.2
		40p	3.1	3.6	0.5	0.6	4.6	4.1	1.0	1.1	1.2	4.4	3.3	4.1	0.5	0.6	4.7	4.7	2.2	2.3	2.4	2.4	3.9	4.9	0.0	0.0	5.0	4.9	5.0	3.4	3.5	4.2
5	25	3p	3.2	9.0	3.1	7.2	5.1	4.1	1.9	2.5	4.3	8.5	3.3	7.9	3.1	6.6	4.7	4.7	3.2	4.2	4.5	4.5	3.9	5.6	3.1	6.2	5.0	4.9	5.0	4.7	4.9	5.0
		5p	3.2	6.6	3.1	6.5	5.2	4.4	1.9	2.6	3.6	7.8	3.3	6.2	3.0	6.2	4.9	4.9	3.4	4.4	4.3	4.3	4.0	5.0	3.1	6.4	5.0	5.0	5.0	4.8	4.9	5.0
		10p	3.2	5.1	3.1	6.1	5.2	4.7	2.0	2.7	3.1	6.5	3.4	5.1	3.0	6.1	5.0	5.0	3.5	4.5	4.2	4.2	4.2	5.0	3.1	6.6	5.0	5.0	5.0	4.8	4.9	5.0
		20p	3.2	4.9	3.1	6.0	5.1	4.9	2.1	2.8	2.9	5.2	3.5	5.0	3.0	6.1	5.0	5.0	3.6	4.6	4.1	4.1	4.3	5.0	3.1	6.5	5.0	5.0	5.0	4.9	4.9	5.0
		40p	3.3	4.9	3.1	5.8	5.0	5.0	2.1	2.8	2.8	5.0	3.6	5.0	3.1	6.0	5.0	5.0	3.6	4.6	4.1	4.1	4.4	5.0	3.2	4.7	5.0	5.0	5.0	4.9	4.9	5.0
5	50	3p	3.3	21.2	3.0	11.3	5.1	5.0	4.3	5.0	5.9	14.7	3.8	16.9	3.0	7.3	5.0	5.0	5.0	5.0	5.0	5.0	4.6	11.5	3.0	7.0	5.0	5.0	5.0	4.9	5.0	5.0
		5p	3.4	14.7	3.0	8.0	5.0	5.0	4.4	5.0	5.0	12.9	3.9	12.7	3.0	7.2	5.0	5.0	5.0	5.0	5.0	5.0	4.9	7.2	3.0	7.2	5.0	5.0	5.0	5.0	5.0	5.0
		10p	3.6	8.3	3.0	7.4	5.0	5.0	4.5	5.0	5.0	9.4	4.1	8.3	3.0	7.3	5.0	5.0	5.0	5.0	5.0	5.0	5.2	5.0	3.0	7.3	5.0	5.0	5.0	4.9	5.0	5.0
		20p	3.9	5.2	3.0	7.4	5.0	5.0	4.5	5.0	5.0	5.8	4.3	5.5	3.0	7.4	5.0	5.0	5.0	5.0	5.0	5.0	5.3	5.0	3.0	7.3	5.0	5.0	5.0	4.9	5.0	5.0
		40p	4.1	5.0	3.0	7.4	5.0	5.0	4.4	5.0	5.0	5.0	4.5	5.0	3.0	7.4	5.0	5.0	5.0	5.0	5.0	5.0	5.4	5.0	3.0	7.3	5.0	5.0	5.0	5.0	5.0	5.0
7	21	3p	3.4	7.8	3.2	6.5	5.6	4.0	1.2	1.6	4.0	7.7	3.4	7.6	3.2	6.9	5.7	5.7	2.7	3.0	4.5	4.5	3.3	6.7	3.1	7.2	5.8	5.4	5.8	4.0	5.0	5.7
		5p	3.3	6.2	3.2	6.2	6.0	4.5	1.2	1.6	3.1	7.4	3.5	6.7	3.2	6.9	6.2	6.2	3.0	3.2	4.2	4.2	3.2	6.5	3.1	7.0	6.2	5.9	6.2	4.4	4.9	5.7
		10p	3.3	5.4	3.1	5.9	6.3	5.0	1.2	1.5	2.4	6.8	3.5	6.3	3.2	6.7	6.5	6.5	3.1	3.3	3.9	3.9	3.1	6.5	3.1	5.9	6.5	6.3	6.5	4.7	4.8	5.7
		20p	3.3	5.4	3.1	5.3	6.5	5.4	1.3	1.5	2.0	6.4	3.6	6.2	3.3	5.9	6.6	6.6	3.2	3.3	3.8	3.8	3.1	6.5	3.0	4.1	6.6	6.4	6.6	4.8	4.8	5.7
		40p	3.3	5.6	3.0	4.2	6.6	5.8	1.3	1.5	1.9	6.2	3.7	6.2	3.3	5.0	6.7	6.7	3.3	3.3	3.7	3.7	3.0	6.5	3.0	3.5	6.8	6.5	6.8	4.9	4.8	5.7
7	35	3p	3.5	14.4	3.1	10.7	6.9	5.8	2.9	3.8	6.1	11.8	3.2	12.2	3.1	9.2	6.6	6.6	4.8	6.1	6.4	6.4	3.5	8.9	3.1	9.2	6.9	6.8	6.9	6.2	6.8	7.0
		5p	3.5	10.6	3.1	10.1	7.1	6.2	3.0	4.1	5.2	10.9	3.2	9.8	3.0	8.8	6.8	6.8	5.0	6.3	6.2	6.2	3.5	7.2	3.0	9.6	7.0	7.0	7.0	6.3	6.8	7.0
		10p	3.6	7.6	3.0	9.7	7.2	6.6	3.1	4.2	4.7	9.1	3.2	7.6	3.0	8.9	6.9	6.9	5.1	6.4	6.0	6.0	3.5	7.0	3.0	9.8	7.0	7.0	7.0	6.4	6.7	7.0
		20p	3.7	6.8	3.0	9.5	7.0	6.8	3.1	4.3	4.5	7.2	3.2	7.0	3.0	9.0	6.9	6.9	5.1	6.3	5.9	5.9	3.5	7.0	3.0	9.8	7.0	7.0	7.0	6.3	6.7	7.0
		40p	3.8	6.8	3.0	9.2	6.9	6.8	3.1	4.3	4.4	6.8	3.1	6.9	3.0	8.9	7.0	7.0	5.1	6.3	5.8	5.8	3.5	7.0	3.0	8.3	7.0	7.0	7.0	6.3	6.6	7.0
7	70	3p	3.0	33.1	3.0	20.4	7.0	7.0	6.0	7.0	8.2	20.5	3.1	26.3	3.0	11.1	7.0	7.0	6.9	7.0	7.0	7.0	3.3	19.4	3.0	10.2	7.0	7.0	7.0	6.9	7.0	7.0
		5p	3.0	24.1	3.0	13.0	7.0	7.0	6.1	7.0	7.0	17.9	3.1	20.7	3.0	10.7	7.0	7.0	7.0	7.0	7.0	7.0	3.2	12.9	3.0	10.5	7.0	7.0	7.0	7.0	7.0	7.0
		10p	3.0	14.7	3.0	11.2	7.0	7.0	6.0	7.0	7.0	13.1	3.1	14.6	3.0	10.8	7.0	7.0	7.0	7.0	7.0	7.0	3.2	7.5	3.0	10.6	7.0	7.0	7.0	7.0	7.0	7.0
		20p	3.0	8.7	3.0	11.1	7.0	7.0	5.9	7.0	7.0	7.9	3.1	9.8	3.0	10.9	7.0	7.0	6.9	7.0	7.0	7.0	3.1	7.0	3.0	10.6	7.0	7.0	7.0	7.0	7.0	7.0
		40p	3.0	7.0	3.0	11.0	7.0	7.0	5.8	7.0	7.0	7.0	3.1	7.1	3.0	11.0	7.0	7.0	6.9	7.0	7.0	7.0	3.0	7.0	3.0	10.7	7.0	7.0	7.0	7.0	7.0	7.0

Table 3
Mean Number of Factors Retained, Phi = 0.30

Low Communality

k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	3.1	2.1	0.0	0.0	1.8	1.5	0.7	1.1	1.5	3.2
		5p	3.0	1.6	0.0	0.0	1.9	1.5	0.8	1.0	1.1	3.0
		10p	3.0	1.3	0.0	0.0	2.0	1.5	0.9	1.0	1.0	2.6
		20p	3.0	1.3	0.0	0.0	2.2	1.7	1.0	1.0	1.0	2.3
		40p	3.0	1.3	0.0	0.0	2.3	1.9	1.0	1.0	1.0	2.2
3	15	3p	3.0	4.1	3.0	4.8	2.5	2.0	1.0	1.4	2.3	4.9
		5p	3.0	3.0	3.0	4.3	2.7	2.1	1.0	1.4	1.7	4.4
		10p	3.0	2.5	3.0	3.9	2.8	2.4	1.0	1.3	1.4	3.7
		20p	3.0	2.5	2.3	2.9	2.9	2.6	1.0	1.3	1.2	3.0
		40p	3.0	2.6	0.4	0.6	3.0	2.7	1.0	1.3	1.2	2.9
3	30	3p	3.0	10.2	3.0	11.7	3.1	2.8	1.7	2.6	3.6	8.9
		5p	3.0	6.5	3.0	9.2	3.1	3.0	1.8	2.7	2.9	7.8
		10p	3.0	3.6	3.0	6.2	3.1	3.0	1.9	2.8	2.8	5.8
		20p	3.0	3.0	3.0	4.8	3.0	3.0	2.0	2.9	2.6	3.6
		40p	3.0	3.0	3.0	4.5	3.0	3.0	2.0	2.9	2.5	3.0
5	15	3p	3.0	4.2	3.0	5.1	2.4	1.7	1.0	1.1	2.2	5.2
		5p	3.0	2.8	3.0	4.7	2.5	1.7	1.0	1.1	1.4	4.9
		10p	3.0	2.0	3.0	4.2	2.9	1.9	1.0	1.0	1.0	4.3
		20p	3.0	1.9	2.7	3.6	3.4	2.3	1.0	1.0	1.0	3.7
		40p	3.0	2.0	1.2	1.5	3.8	2.7	1.0	1.0	1.0	3.3
5	25	3p	3.0	8.6	3.0	11.2	3.4	2.4	1.0	1.5	3.6	8.2
		5p	3.0	6.0	3.0	10.1	3.9	2.8	1.0	1.5	2.4	7.5
		10p	3.0	4.5	3.0	9.2	4.7	3.5	1.0	1.4	1.7	6.3
		20p	3.0	4.4	3.0	8.4	5.0	4.2	1.0	1.4	1.4	5.2
		40p	3.0	4.5	3.0	7.4	5.1	4.7	1.0	1.4	1.3	4.9
5	50	3p	3.0	20.8	3.0	26.9	4.5	4.0	2.1	4.1	5.7	14.4
		5p	3.0	14.4	3.0	24.7	4.8	4.6	2.2	4.3	4.6	12.6
		10p	3.0	8.1	3.0	22.3	5.0	4.9	2.3	4.4	4.2	9.2
		20p	3.0	5.2	3.0	19.9	5.0	5.0	2.4	4.5	4.0	5.6
		40p	3.0	5.0	3.0	17.3	5.0	5.0	2.5	4.5	3.9	5.0
7	21	3p	3.0	7.0	3.0	8.7	3.2	2.1	1.0	1.2	3.1	7.3
		5p	3.0	4.9	3.0	8.1	3.6	2.2	1.0	1.1	1.9	6.9
		10p	3.0	3.6	3.0	7.3	4.5	2.8	1.0	1.0	1.1	6.1
		20p	3.0	3.4	3.0	6.7	5.4	3.5	1.0	1.0	1.0	5.4
		40p	3.0	3.6	3.0	5.7	5.8	4.2	1.0	1.0	1.0	5.0
7	35	3p	3.0	13.6	3.0	17.7	4.0	3.0	1.1	2.1	4.8	11.0
		5p	3.0	9.7	3.0	16.2	5.0	3.7	1.1	2.1	3.4	10.0
		10p	3.0	6.8	3.0	14.7	6.0	4.8	1.1	2.1	2.5	8.2
		20p	3.0	6.1	3.0	13.2	6.6	5.5	1.1	2.1	2.1	6.6
		40p	3.0	6.1	3.0	11.6	6.8	6.0	1.1	2.1	1.9	6.3
7	70	3p	3.0	32.7	3.0	39.3	6.1	5.5	2.9	6.0	8.0	20.1
		5p	3.0	23.6	3.0	36.2	6.8	6.5	3.0	6.3	6.6	17.6
		10p	3.0	14.3	3.0	33.0	7.0	6.9	3.0	6.4	6.2	12.8
		20p	3.0	8.6	3.0	29.9	7.0	7.0	3.0	6.5	5.9	7.8
		40p	3.0	7.0	3.0	26.7	7.0	7.0	3.1	6.5	5.7	7.0

Wide Communality

k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	3.0	2.2	0.0	0.0	1.8	1.8	1.0	1.2	1.5	1.5
		5p	3.0	1.9	0.0	0.0	2.0	2.0	1.0	1.2	1.3	1.3
		10p	3.0	1.8	0.0	0.0	2.4	2.4	1.0	1.2	1.1	1.1
		20p	3.0	1.8	0.0	0.0	2.7	2.7	1.0	1.2	1.1	1.1
		40p	3.0	1.8	0.0	0.0	2.8	2.8	1.0	1.2	1.0	1.0
3	15	3p	3.0	3.8	3.0	4.6	2.5	2.5	1.3	2.2	2.4	2.4
		5p	3.0	3.2	3.0	4.3	2.8	2.8	1.4	2.3	2.2	2.2
		10p	3.0	2.9	2.9	3.9	3.0	3.0	1.5	2.4	2.1	2.1
		20p	3.0	3.0	2.0	2.6	3.0	3.0	1.7	2.4	2.0	2.0
		40p	3.0	3.0	0.4	0.5	3.0	3.0	1.7	2.4	2.0	2.0
3	30	3p	3.0	8.1	3.0	9.2	3.0	3.0	2.7	3.0	3.0	3.0
		5p	3.0	5.6	3.0	6.2	3.0	3.0	2.8	3.0	3.0	3.0
		10p	3.0	3.4	3.0	4.8	3.0	3.0	2.9	3.0	3.0	3.0
		20p	3.0	3.0	3.0	4.7	3.0	3.0	3.0	3.0	3.0	3.0
		40p	3.0	3.0	3.0	4.7	3.0	3.0	3.0	3.0	3.0	3.0
5	15	3p	3.0	4.3	3.0	5.2	2.6	2.6	1.1	1.7	2.4	2.4
		5p	3.0	3.5	3.0	4.8	3.1	3.1	1.0	1.7	1.9	1.9
		10p	3.0	3.2	3.0	4.2	3.7	3.7	1.0	1.8	1.6	1.6
		20p	3.0	3.3	2.4	3.1	4.1	4.1	1.0	1.8	1.4	1.4
		40p	3.0	3.3	1.0	1.3	4.3	4.3	1.0	1.8	1.3	1.3
5	25	3p	3.0	7.5	3.0	11.3	3.6	3.6	1.6	3.3	3.7	3.7
		5p	3.0	6.0	3.0	10.3	4.3	4.3	1.7	3.5	3.2	3.2
		10p	3.0	5.0	3.0	9.3	4.9	4.9	1.9	3.7	3.0	3.0
		20p	3.0	5.0	3.0	8.2	5.0	5.0	2.0	3.8	2.8	2.8
		40p	3.0	5.0	3.0	7.0	5.0	5.0	2.1	3.8	2.7	2.7
5	50	3p	3.0	16.8	3.0	27.0	5.0	5.0	3.9	5.0	5.0	5.0
		5p	3.0	12.7	3.0	24.6	5.0	5.0	4.2	5.0	5.0	5.0
		10p	3.0	8.4	3.0	21.2	5.0	5.0	4.4	5.0	5.0	5.0
		20p	3.0	5.6	3.0	17.6	5.0	5.0	4.4	5.0	5.0	5.0
		40p	3.0	5.0	3.0	14.7	5.0	5.0	4.4	5.0	5.0	5.0
7	21	3p	3.0	6.7	3.0	9.0	3.4	3.4	1.3	2.3	3.3	3.3
		5p	3.0	5.7	3.0	8.3	4.4	4.4	1.3	2.4	2.7	2.7
		10p	3.0	5.2	3.0	7.5	5.5	5.5	1.4	2.4	2.2	2.2
		20p	3.0	5.2	3.0	6.7	6.0	6.0	1.5	2.5	2.0	2.0
		40p	3.0	5.4	3.0	5.4	6.2	6.2	1.6	2.5	1.9	1.9
7	35	3p	3.0	12.1	3.0	17.9	5.1	5.1	2.1	4.5	5.2	5.2
		5p	3.0	9.7	3.0	16.4	6.0	6.0	2.3	4.8	4.6	4.6
		10p	3.0	7.6	3.0	14.6	6.8	6.8	2.4	5.0	4.2	4.2
		20p	3.0	7.0	3.0	13.1	7.0	7.0	2.5	5.2	4.0	4.0
		40p	3.0	7.0	3.0	11.4	7.0	7.0	2.6	5.3	3.9	3.9
7	70	3p	3.0	26.3	3.0	39.4	6.9	6.9	5.2	7.0	7.0	7.0
		5p	3.0	20.7	3.0	36.2	7.0	7.0	5.6	7.0	7.0	7.0
		10p	3.0	14.5	3.0	32.7	7.0	7.0	5.7	7.0	6.9	6.9
		20p	3.0	9.7	3.0	29.4	7.0	7.0	5.6	7.0	6.9	6.9
		40p	3.0	7.1	3.0	26.2	7.0	7.0	5.5	7.0	6.9	6.9

High Communality

k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	3.0	2.5	0.0	0.0	2.1	2.0	2.1	1.9	1.9	2.2
		5p	3.0	2.5	0.0	0.0	2.4	2.3	2.4	2.0	1.8	2.2
		10p	3.0	2.5	0.0	0.0	2.7	2.5	2.7	2.1	1.8	2.2
		20p	3.0	2.6	0.0	0.0	2.8	2.6	2.8	2.1	1.7	2.3
		40p	3.0	2.6	0.0	0.0	2.9	2.7	2.9	2.2	1.7	2.2
3	15	3p	3.0	3.1	3.0	4.3	2.8	2.7	2.8	2.9	2.7	2.9
		5p	3.0	3.0	3.0	4.2	2.9	2.9	2.9	3.0	2.7	2.9
		10p	3.0	3.0	2.5	3.4	3.0	3.0	3.0	3.0	2.7	2.9
		20p	3.0	3.0	0.5	0.6	3.0	3.0	3.0	3.0	2.7	2.9
		40p	3.0	3.0	0.0	0.0	3.0	3.0	3.0	3.0	2.7	2.9
3	30	3p	3.0	4.8	3.0	4.5	3.0	3.0	3.0	3.0	3.0	3.0
		5p	3.0	3.2	3.0	4.4	3.0	3.0	3.0	3.0	3.0	3.0
		10p	3.0	3.0	3.0	4.4	3.0	3.0	3.0	3.0	3.0	3.0
		20p	3.0	3.0	3.0	4.4	3.0	3.0	3.0	3.0	3.0	3.0
		40p	3.0	3.0	3.0	4.4	3.0	3.0	3.0	3.0	3.0	3.0
5	15	3p	3.0	4.2	3.0	5.5	2.9	2.6	2.9	2.7	2.7	3.3
		5p	3.0	4.1	3.0	4.9	3.6	3.2	3.6	2.9	2.5	3.3
		10p	3.0	4.2	2.6	3.5	4.2	3.8	4.2	3.1	2.4	3.3
		20p	3.0	4.3	0.5	0.7	4.5	4.2	4.5	3.1	2.3	3.2
		40p	3.0	4.3	0.0	0.0	4.7	4.4	4.7	3.2	2.2	3.2
5	25	3p	3.0	5.5	3.0	10.9	4.1	3.8	4.1	4.7	4.0	4.6
		5p	3.0	5.0	3.0	10.0	4.7	4.5	4.7	4.8	3.9	4.6
		10p	3.0	4.9	3.0	8.8	4.9	4.9	4.9	4.9	3.8	4.6
		20p	3.0	4.9	3.0	7.3	4.9	4.9	4.9	4.9	3.8	4.7
		40p	3.0	4.9	3.0	5.4	5.0	4.9	5.0	4.9	3.8	4.6
5	50	3p	3.0	11.4	3.0	23.8	5.0	4.9	5.0	5.0	5.0	5.0
		5p	3.0	7.2	3.0	19.5	5.0	5.0	5.0	5.0	5.0	5.0
		10p	3.0	5.0	3.0	15.2	5.0	5.0	5.0	5.0	5.0	5.0
		20p	3.0	5.0	3.0	13.3	5.0	5.0	5.0	5.0	5.0	5.0
		40p	3.0	5.0	3.0	12.4	5.0	5.0	5.0	5.0	5.0	5.0
7	21	3p	3.0	6.3	3.0	9.4	4.2	3.6	4.2	3.4	3.7	4.7
		5p	3.0	6.1	3.0	8.4	5.2	4.6	5.2	3.7	3.6	4.7
		10p	3.0	6.1	3.0	6.9	6.0	5.4	6.0	3.9	3.4	4.7
		20p	3.0	6.2	3.0	5.1	6.4	5.9	6.4	4.0	3.3	4.6
		40p	3.0	6.3	3.0	3.9	6.7	6.2	6.7	4.2	3.3	4.7
7	35	3p	3.0	8.7	3.0	17.7	5.7	5.3	5.7	6.4	5.6	6.4
		5p	3.0	7.2	3.0	16.0	6.5	6.2	6.5	6.6	5.4	6.4
		10p	3.0	7.0	3.0	13.9	6.8	6.7	6.8	6.7	5.3	6.5
		20p	3.0	7.0	3.0	11.5	7.0	6.9	7.0	6.8	5.3	6.5
		40p	3.0	7.0	3.0	8.9	7.0	7.0	7.0	6.8	5.2	6.5
7	70	3p	3.0	19.6	3.0	39.1	6.9	6.9	6.9	7.0	7.0	7.0
		5p	3.0	13.0	3.0	35.7	7.0	7.0	7.0	7.0	7.0	7.0
		10p	3.0	7.6	3.0	31.9	7.0	7.0	7.0	7.0	7.0	7.0
		20p	3.0	7.0	3.0	28.4	7.0	7.0	7.0	7.0	7.0	7.0
		40p	3.0	7.0	3.0	24.7	7.0	7.0	7.0	7.0	7.0	7.0

Table 4
Mean Number of Factors Retained, Phi = 0.50

			Low Communality										Wide Communality										High Communality									
k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	3.1	2.0	0.0	0.0	1.7	1.4	0.8	1.1	1.4	3.0	3.0	1.9	0.0	0.0	1.5	1.5	1.0	1.2	1.3	1.3	3.0	2.1	0.0	0.0	1.6	1.4	1.6	1.6	1.4	1.9
		5p	3.0	1.4	0.0	0.0	1.6	1.3	0.9	1.0	1.1	2.8	3.0	1.6	0.0	0.0	1.6	1.6	1.0	1.2	1.1	1.1	3.0	2.0	0.0	0.0	1.8	1.6	1.8	1.6	1.3	1.8
		10p	3.0	1.2	0.0	0.0	1.7	1.3	1.0	1.0	1.0	2.4	3.0	1.4	0.0	0.0	1.9	1.9	1.0	1.2	1.0	1.0	3.0	2.0	0.0	0.0	2.2	2.0	2.2	1.7	1.2	1.7
		20p	3.0	1.1	0.0	0.0	1.8	1.4	1.0	1.0	1.0	2.0	3.0	1.4	0.0	0.0	2.2	2.2	1.0	1.1	1.0	1.0	3.0	2.1	0.0	0.0	2.5	2.3	2.5	1.7	1.1	1.7
		40p	3.0	1.1	0.0	0.0	1.9	1.5	1.0	1.0	1.0	1.8	3.0	1.4	0.0	0.0	2.5	2.5	1.0	1.1	1.0	1.0	3.0	2.1	0.0	0.0	2.7	2.4	2.7	1.7	1.1	1.7
3	15	3p	3.0	3.9	3.0	5.2	2.0	1.5	1.0	1.1	2.0	4.8	3.0	3.6	3.0	5.1	1.9	1.9	1.1	1.8	2.0	2.0	3.0	3.0	3.0	4.9	2.1	1.9	2.1	2.7	2.1	2.6
		5p	3.0	2.5	3.0	4.7	2.0	1.5	1.0	1.1	1.2	4.4	3.0	2.9	3.0	4.6	2.3	2.3	1.1	1.9	1.7	1.7	3.0	3.0	3.0	4.5	2.6	2.4	2.6	2.8	2.0	2.6
		10p	3.0	1.9	3.0	4.3	2.4	1.7	1.0	1.0	1.0	3.7	3.0	2.6	3.0	4.1	2.7	2.7	1.1	1.9	1.4	1.4	3.0	3.0	2.7	3.7	2.9	2.8	2.9	2.8	1.9	2.6
		20p	3.0	1.8	2.6	3.5	2.7	2.0	1.0	1.0	1.0	3.0	3.0	2.7	2.3	2.9	2.9	2.9	1.1	2.0	1.4	1.4	3.0	3.0	0.7	0.9	3.0	3.0	3.0	2.9	1.9	2.6
		40p	3.0	1.9	1.0	1.3	2.9	2.4	1.0	1.0	1.0	2.7	3.0	2.7	0.7	0.9	3.0	3.0	1.1	2.0	1.3	1.3	3.0	3.0	0.0	0.0	3.0	3.0	3.0	2.9	1.9	2.6
3	30	3p	3.0	10.0	3.0	14.3	2.5	2.1	1.1	2.0	3.3	8.5	3.0	8.1	3.0	13.5	2.8	2.8	1.8	3.0	3.0	3.0	3.0	4.7	3.0	8.1	3.0	2.9	3.0	3.0	3.0	3.0
		5p	3.0	6.3	3.0	12.6	2.8	2.5	1.1	2.1	2.5	7.4	3.0	5.6	3.0	10.9	2.9	2.9	2.0	3.0	2.9	2.9	3.0	3.2	3.0	5.7	3.0	3.0	3.0	3.0	3.0	3.0
		10p	3.0	3.5	3.0	10.4	3.0	2.9	1.1	2.2	2.1	5.4	3.0	3.5	3.0	7.1	3.0	3.0	2.2	3.0	2.8	2.8	3.0	3.0	3.0	5.0	3.0	3.0	3.0	3.0	3.0	3.0
		20p	3.0	3.0	3.0	8.0	3.0	3.0	1.1	2.2	1.9	3.3	3.0	3.0	3.0	5.3	3.0	3.0	2.3	3.0	2.8	2.8	3.0	3.0	3.0	4.9	3.0	3.0	3.0	3.0	3.0	3.0
		40p	3.0	3.0	3.0	6.1	3.0	3.0	1.1	2.2	1.7	3.0	3.0	3.0	3.0	4.8	3.0	3.0	2.4	3.0	2.7	2.7	3.0	3.0	3.0	4.9	3.0	3.0	3.0	3.0	3.0	3.0
5	15	3p	3.0	3.9	3.0	5.2	2.0	1.5	1.0	1.1	2.0	5.0	3.0	3.7	3.0	5.5	1.6	1.6	1.0	1.4	1.8	1.8	3.0	3.6	3.0	5.5	2.0	1.7	2.0	2.3	2.0	2.7
		5p	3.0	2.5	3.0	4.7	2.0	1.4	1.0	1.0	1.2	4.6	3.0	3.0	3.0	5.0	2.0	2.0	1.0	1.3	1.3	1.3	3.0	3.5	3.0	4.9	2.7	2.2	2.7	2.5	1.8	2.6
		10p	3.0	1.6	3.0	4.2	2.2	1.4	1.0	1.0	1.0	4.0	3.0	2.6	3.0	4.3	2.9	2.9	1.0	1.3	1.0	1.0	3.0	3.5	2.8	3.8	3.4	3.0	3.4	2.6	1.6	2.5
		20p	3.0	1.4	2.8	3.6	2.7	1.6	1.0	1.0	1.0	3.2	3.0	2.6	2.5	3.3	3.6	3.6	1.0	1.2	1.0	1.0	3.0	3.6	1.0	1.2	3.8	3.5	3.8	2.7	1.5	2.5
		40p	3.0	1.3	1.5	1.9	3.3	1.9	1.0	1.0	1.0	2.7	3.0	2.7	1.3	1.6	4.0	4.0	1.0	1.2	1.0	1.0	3.0	3.6	0.0	0.1	4.1	3.7	4.1	2.7	1.4	2.5
5	25	3p	3.0	7.9	3.0	11.6	1.9	1.4	1.0	1.2	2.9	7.6	3.0	7.1	3.0	11.9	2.4	2.4	1.1	2.6	3.0	3.0	3.0	5.4	3.0	11.8	2.6	2.2	2.6	4.1	3.0	3.8
		5p	3.0	5.2	3.0	10.5	2.2	1.5	1.0	1.1	1.6	6.8	3.0	5.6	3.0	10.8	3.3	3.3	1.1	2.7	2.4	2.4	3.0	4.9	3.0	10.6	3.7	3.1	3.7	4.3	2.7	3.7
		10p	3.0	3.3	3.0	9.3	3.0	2.0	1.0	1.1	1.1	5.4	3.0	4.7	3.0	9.6	4.3	4.3	1.1	2.9	2.1	2.1	3.0	4.9	3.0	9.2	4.6	4.2	4.6	4.5	2.5	3.7
		20p	3.0	3.0	3.0	8.3	3.8	2.6	1.0	1.1	1.0	4.2	3.0	4.7	3.0	8.5	4.8	4.8	1.1	2.9	1.9	1.9	3.0	5.0	3.0	7.6	4.9	4.7	4.9	4.5	2.4	3.7
		40p	3.0	3.1	3.0	7.5	4.4	3.2	1.0	1.0	1.0	3.7	3.0	4.7	3.0	7.2	4.9	4.9	1.1	3.0	1.7	1.7	3.0	5.0	3.0	5.8	5.0	4.9	5.0	4.5	2.4	3.7
5	50	3p	3.0	20.7	3.0	27.1	2.9	2.2	1.1	2.7	5.4	14.2	3.0	16.3	3.0	27.2	4.0	4.0	2.1	4.8	4.8	4.8	3.0	11.2	3.0	27.0	4.6	4.4	4.6	5.0	4.9	5.0
		5p	3.0	14.2	3.0	25.0	3.9	3.1	1.0	2.9	3.7	12.5	3.0	12.2	3.0	25.0	4.6	4.6	2.4	4.9	4.4	4.4	3.0	7.0	3.0	24.6	5.0	4.9	5.0	5.0	4.9	5.0
		10p	3.0	8.0	3.0	22.8	4.8	4.3	1.0	3.0	2.8	9.1	3.0	8.0	3.0	22.5	5.0	5.0	2.7	5.0	4.1	4.1	3.0	5.0	3.0	21.9	5.0	5.0	5.0	5.0	4.9	5.0
		20p	3.0	5.2	3.0	20.7	4.9	4.8	1.0	3.1	2.3	5.6	3.0	5.4	3.0	20.2	5.0	5.0	2.8	5.0	4.0	4.0	3.0	5.0	3.0	19.5	5.0	5.0	5.0	5.0	4.9	5.0
		40p	3.0	5.0	3.0	18.6	5.0	4.9	1.0	3.1	2.1	5.0	3.0	5.0	3.0	18.0	5.0	5.0	2.9	5.0	3.9	3.9	3.0	5.0	3.0	17.0	5.0	5.0	5.0	5.0	4.9	5.0
7	21	3p	3.0	6.3	3.0	9.0	1.8	1.3	1.0	1.1	2.5	6.6	3.0	6.1	3.0	9.4	1.8	1.8	1.0	1.5	2.4	2.4	3.0	5.4	3.0	9.5	1.9	1.6	1.9	2.6	2.4	3.5
		5p	3.0	4.0	3.0	8.2	1.9	1.3	1.0	1.0	1.3	6.0	3.0	4.9	3.0	8.6	2.5	2.5	1.0	1.4	1.6	1.6	3.0	5.2	3.0	8.6	3.1	2.4	3.1	2.7	2.0	3.3
		10p	3.0	2.5	3.0	7.4	2.6	1.5	1.0	1.0	1.0	5.0	3.0	4.2	3.0	7.7	4.1	4.1	1.0	1.3	1.1	1.1	3.0	5.3	3.0	7.3	4.8	3.8	4.8	2.9	1.7	3.2
		20p	3.0	2.1	3.0	6.7	3.7	2.0	1.0	1.0	1.0	4.0	3.0	4.2	3.0	6.9	5.3	5.3	1.0	1.3	1.0	1.0	3.0	5.4	3.0	5.7	5.7	4.8	5.7	3.0	1.4	3.1
		40p	3.0	2.2	3.0	5.9	4.7	2.6	1.0	1.0	1.0	3.4	3.0	4.4	3.0	6.0	6.0	6.0	1.0	1.2	1.0	1.0	3.0	5.5	3.0	4.4	6.2	5.4	6.2	3.0	1.3	3.1
7	35	3p	3.0	13.0	3.0	17.9	1.8	1.3	1.0	1.3	4.1	10.6	3.0	11.7	3.0	18.1	2.7	2.7	1.0	3.0	4.1	4.1	3.0	8.4	3.0	17.9	3.5	3.0	3.5	5.5	4.2	5.2
		5p	3.0	9.0	3.0	16.5	2.5	1.5	1.0	1.2	2.1	9.6	3.0	9.2	3.0	16.6	4.4	4.4	1.0	3.2	3.1	3.1	3.0	7.0	3.0	16.2	4.9	4.3	4.9	5.8	3.9	5.1
		10p	3.0	5.9	3.0	15.0	4.4	2.4	1.0	1.1	1.1	7.8	3.0	7.1	3.0	14.9	6.0	6.0	1.0	3.3	2.4	2.4	3.0	6.8	3.0	14.2	6.2	5.6	6.2	5.9	3.7	5.1
		20p	3.0	5.2	3.0	13.7	6.0	4.0	1.0	1.1	1.1	6.2	3.0	6.5	3.0	13.4	6.5	6.5	1.0	3.3	1.9	1.9	3.0	6.8	3.0	12.0	6.8	6.4	6.8	6.0	3.6	5.1
		40p	3.0	5.4	3.0	12.3	6.6	5.0	1.0	1.1	1.0	5.6	3.0	6.5	3.0	11.9	6.8	6.8	1.0	3.4	1.8	1.8	3.0	6.8	3.0	9.5	6.9	6.8	6.9	6.0	3.5	5.1
7	70	3p	3.0	32.4	3.0	39.4	3.2	2.4	1.1	3.7	7.5	19.8	3.0	25.9	3.0	39.6	5.2	5.2	2.6	6.6	6.6	6.6	3.0	19.4	3.0	39.3	6.2	5.9	6.2	7.0	6.8	7.0
		5p	3.0	23.4	3.0	36.5	5.0	3.9	1.1	3.8	5.1	17.3	3.0	20.4	3.0	36.5	6.3	6.3	3.0	6.7	6.1	6.1	3.0	12.9	3.0	36.1	6.9	6.7	6.9	7.0	6.8	7.0
		10p	3.0	14.1	3.0	33.5	6.7	5.9	1.1	4.0	3.8	12.7	3.0	14.3	3.0	33.2	6.9	6.9	3.2	6.8	5.8	5.8	3.0	7.5	3.0	32.5	7.0	7.0	7.0	7.0	6.7	7.0
		20p	3.0	8.5	3.0	30.7	7.0	6.8	1.2	4.0	3.1	7.7	3.0	9.6	3.0	30.2	7.0	7.0	3.3	6.8	5.7	5.7	3.0	7.0	3.0	29.1	7.0	7.0	7.0	7.0	6.7	7.0
		40p	3.0	7.0	3.0	27.9	7.0	7.0	1.3	4.0	2.8	7.0	3.0	7.1	3.0	27.1	7.0	7.0	3.4	6.8	5.7	5.7	3.0	7.0	3.0	25.5	7.0	7.0	7.0	7.0	6.7	7.0

Table 5
Mean Number of Factors Retained, Phi = Mixed

Low Communality

k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	3.1	2.1	0.0	0.0	1.8	1.4	0.7	1.1	1.5	3.2
		5p	3.0	1.5	0.0	0.0	1.8	1.4	0.8	1.0	1.1	3.0
		10p	3.0	1.3	0.0	0.0	1.9	1.4	0.9	1.0	1.0	2.6
		20p	3.0	1.2	0.0	0.0	2.1	1.6	1.0	1.0	1.0	2.2
		40p	3.0	1.2	0.0	0.0	2.2	1.7	1.0	1.0	1.0	2.0
3	15	3p	3.1	4.3	3.1	4.5	2.9	2.3	1.1	1.5	2.5	5.0
		5p	3.0	3.2	3.0	4.1	3.0	2.5	1.1	1.5	2.0	4.6
		10p	3.0	2.7	3.0	3.8	3.0	2.7	1.1	1.6	1.7	3.9
		20p	3.0	2.7	2.2	2.7	3.0	2.8	1.1	1.6	1.6	3.1
		40p	3.0	2.8	0.3	0.4	3.0	2.9	1.2	1.6	1.5	3.0
3	30	3p	3.0	10.2	3.0	7.0	2.9	2.6	1.9	2.5	3.5	8.8
		5p	3.0	6.5	3.0	4.9	3.0	2.8	2.0	2.5	2.8	7.7
		10p	3.0	3.6	3.0	3.7	3.0	3.0	2.0	2.5	2.4	5.6
		20p	3.0	3.0	3.0	3.6	3.0	3.0	2.0	2.6	2.3	3.5
		40p	3.0	3.0	3.0	3.6	3.0	3.0	2.0	2.6	2.2	3.0
5	15	3p	3.0	4.2	3.0	5.0	2.4	1.8	1.0	1.2	2.2	5.1
		5p	3.0	2.9	3.0	4.6	2.5	1.8	1.0	1.1	1.5	4.7
		10p	3.0	2.2	3.0	4.1	2.8	2.1	1.0	1.1	1.1	4.1
		20p	3.0	2.1	2.6	3.3	3.2	2.4	1.0	1.1	1.1	3.4
		40p	3.0	2.1	1.1	1.3	3.4	2.7	1.0	1.1	1.0	3.1
5	25	3p	3.0	8.4	3.0	10.3	3.3	2.5	1.2	1.9	3.5	7.8
		5p	3.0	5.8	3.0	9.1	3.6	2.9	1.2	1.9	2.5	7.1
		10p	3.0	4.1	3.0	7.9	4.1	3.4	1.2	1.9	2.0	5.8
		20p	3.0	3.8	3.0	7.2	4.3	3.7	1.2	1.9	1.8	4.5
		40p	3.0	3.9	3.0	6.5	4.5	3.9	1.2	1.9	1.7	4.0
5	50	3p	3.0	20.8	3.0	27.0	3.5	2.9	1.5	3.2	5.5	14.4
		5p	3.0	14.3	3.0	24.9	4.3	3.7	1.6	3.3	3.9	12.6
		10p	3.0	8.1	3.0	22.5	4.9	4.5	1.7	3.3	3.1	9.3
		20p	3.0	5.2	3.0	20.3	5.0	4.9	1.8	3.4	2.8	5.7
		40p	3.0	5.0	3.0	17.9	5.0	5.0	1.9	3.3	2.7	5.0
7	21	3p	3.0	6.8	3.0	8.3	3.1	2.2	1.0	1.4	3.1	7.0
		5p	3.0	4.7	3.0	7.5	3.4	2.4	1.0	1.4	2.0	6.5
		10p	3.0	3.5	3.0	6.7	4.0	2.8	1.0	1.3	1.5	5.6
		20p	3.0	3.3	3.0	6.1	4.6	3.3	1.0	1.3	1.3	4.7
		40p	3.0	3.4	3.0	5.4	4.9	3.7	1.0	1.3	1.2	4.2
7	35	3p	3.0	13.6	3.0	17.6	4.3	3.4	1.3	2.4	4.9	11.1
		5p	3.0	9.6	3.0	16.1	5.0	4.0	1.2	2.5	3.6	10.1
		10p	3.0	6.5	3.0	14.6	5.8	4.7	1.2	2.5	2.9	8.2
		20p	3.0	5.8	3.0	13.0	6.1	5.3	1.2	2.5	2.5	6.3
		40p	3.0	5.9	3.0	11.4	6.2	5.8	1.2	2.6	2.3	6.0
7	70	3p	3.0	32.5	3.0	39.3	4.7	4.2	2.9	4.9	7.6	19.9
		5p	3.0	23.5	3.0	36.2	5.7	5.0	2.9	5.0	5.6	17.3
		10p	3.0	14.2	3.0	32.9	6.7	6.2	3.0	5.1	4.7	12.6
		20p	3.0	8.5	3.0	29.6	7.0	6.8	3.0	5.2	4.3	7.7
		40p	3.0	7.0	3.0	25.9	7.0	7.0	3.0	5.2	4.1	7.0

Wide Communality

k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	3.0	2.3	0.0	0.0	1.9	1.9	1.1	1.4	1.7	1.7
		5p	3.0	2.0	0.0	0.0	2.1	2.1	1.1	1.5	1.4	1.4
		10p	3.0	1.9	0.0	0.0	2.4	2.4	1.1	1.4	1.3	1.3
		20p	3.0	1.9	0.0	0.0	2.6	2.6	1.2	1.4	1.2	1.2
		40p	3.0	1.9	0.0	0.0	2.7	2.7	1.2	1.4	1.2	1.2
3	15	3p	3.0	3.6	3.0	4.3	2.2	2.2	1.3	2.0	2.2	2.2
		5p	3.0	2.9	3.0	3.8	2.4	2.4	1.4	2.1	2.0	2.0
		10p	3.0	2.7	2.9	3.5	2.7	2.7	1.5	2.1	1.9	1.9
		20p	3.0	2.7	1.9	2.2	2.9	2.9	1.6	2.1	1.8	1.8
		40p	3.0	2.8	0.3	0.3	3.0	3.0	1.7	2.1	1.8	1.8
3	30	3p	3.0	8.1	3.0	5.0	2.9	2.9	2.3	3.0	3.0	3.0
		5p	3.0	5.7	3.0	3.9	3.0	3.0	2.4	3.0	2.9	2.9
		10p	3.0	3.5	3.0	3.7	3.0	3.0	2.5	3.0	2.9	2.9
		20p	3.0	3.0	3.0	3.7	3.0	3.0	2.6	3.0	2.9	2.9
		40p	3.0	3.0	3.0	3.7	3.0	3.0	2.6	3.0	2.9	2.9
5	15	3p	3.0	4.2	3.0	4.9	2.5	2.5	1.2	1.8	2.3	2.3
		5p	3.0	3.4	3.0	4.4	2.8	2.8	1.2	1.8	1.9	1.9
		10p	3.0	3.0	3.0	4.0	3.4	3.4	1.2	1.9	1.7	1.7
		20p	3.0	3.0	2.5	3.1	3.9	3.9	1.3	1.9	1.6	1.6
		40p	3.0	3.1	1.0	1.2	4.3	4.3	1.3	1.9	1.5	1.5
5	25	3p	3.0	7.9	3.0	11.0	4.2	4.2	1.9	3.5	4.0	4.0
		5p	3.0	6.2	3.0	10.2	4.7	4.7	2.1	3.8	3.6	3.6
		10p	3.0	5.1	3.0	9.2	4.9	4.9	2.3	4.0	3.3	3.3
		20p	3.0	4.9	3.0	8.2	5.0	5.0	2.4	4.1	3.1	3.1
		40p	3.0	4.9	3.0	7.1	5.0	5.0	2.5	4.1	3.0	3.0
5	50	3p	3.0	16.3	3.0	27.1	4.7	4.7	3.5	5.0	5.0	5.0
		5p	3.0	12.3	3.0	24.7	5.0	5.0	3.7	5.0	4.9	4.9
		10p	3.0	8.1	3.0	21.9	5.0	5.0	3.8	5.0	4.8	4.8
		20p	3.0	5.4	3.0	18.9	5.0	5.0	3.9	5.0	4.8	4.8
		40p	3.0	5.0	3.0	15.9	5.0	5.0	3.9	5.0	4.8	4.8
7	21	3p	3.0	6.4	3.0	8.4	3.3	3.3	1.4	2.5	3.2	3.2
		5p	3.0	5.3	3.0	7.7	3.9	3.9	1.5	2.6	2.7	2.7
		10p	3.0	4.7	3.0	6.9	4.7	4.7	1.5	2.7	2.4	2.4
		20p	3.0	4.6	3.0	6.2	5.3	5.3	1.6	2.8	2.3	2.3
		40p	3.0	4.6	3.0	5.4	5.7	5.7	1.7	2.8	2.3	2.3
7	35	3p	3.0	11.7	3.0	17.7	4.6	4.6	2.4	4.7	4.9	4.9
		5p	3.0	9.3	3.0	16.1	5.5	5.5	2.6	5.0	4.3	4.3
		10p	3.0	7.3	3.0	14.3	6.2	6.2	2.8	5.2	4.0	4.0
		20p	3.0	6.7	3.0	12.7	6.7	6.7	2.9	5.4	3.9	3.9
		40p	3.0	6.7	3.0	11.1	6.9	6.9	3.0	5.4	3.8	3.8
7	70	3p	3.0	26.0	3.0	39.1	5.9	5.9	5.1	6.5	6.4	6.4
		5p	3.0	20.2	3.0	35.2	6.1	6.1	5.2	6.7	6.0	6.0
		10p	3.0	14.0	3.0	29.2	6.7	6.7	5.4	6.8	6.0	6.0
		20p	3.0	9.4	3.0	24.6	6.9	6.9	5.3	6.9	5.9	5.9
		40p	3.0	7.0	3.0	21.1	6.9	6.9	5.3	6.9	5.9	5.9

High Communality

k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	3.0	2.3	0.0	0.0	1.9	1.8	1.9	1.9	1.8	2.1
		5p	3.0	2.2	0.0	0.0	2.2	2.0	2.2	2.0	1.7	2.0
		10p	3.0	2.2	0.0	0.0	2.4	2.2	2.4	2.0	1.7	2.0
		20p	3.0	2.3	0.0	0.0	2.6	2.4	2.6	2.0	1.7	2.0
		40p	3.0	2.3	0.0	0.0	2.8	2.5	2.8	2.0	1.7	2.0
3	15	3p	3.0	3.1	3.0	4.1	2.5	2.4	2.5	2.9	2.4	2.8
		5p	3.0	3.0	3.0	3.9	2.9	2.7	2.9	2.9	2.3	2.8
		10p	3.0	3.0	2.6	3.3	3.0	3.0	3.0	3.0	2.3	2.8
		20p	3.0	3.0	0.6	0.7	3.0	3.0	3.0	3.0	2.2	2.8
		40p	3.0	3.0	0.0	0.0	3.0	3.0	3.0	3.0	2.2	2.8
3	30	3p	3.0	4.7	3.0	4.7	3.0	3.0	3.0	3.0	3.0	3.0
		5p	3.0	3.2	3.0	4.3	3.0	3.0	3.0	3.0	3.0	3.0
		10p	3.0	3.0	3.0	4.3	3.0	3.0	3.0	3.0	3.0	3.0
		20p	3.0	3.0	3.0	4.3	3.0	3.0	3.0	3.0	3.0	3.0
		40p	3.0	3.0	3.0	4.2	3.0	3.0	3.0	3.0	3.0	3.0
5	15	3p	3.0	4.0	3.0	5.0	2.9	2.6	2.9	2.7	2.7	3.2
		5p	3.0	3.8	3.0	4.6	3.3	3.0	3.3	2.8	2.5	3.1
		10p	3.0	3.8	2.7	3.6	3.8	3.5	3.8	2.9	2.5	3.1
		20p	3.0	3.9	0.7	0.9	4.2	3.8	4.2	3.0	2.4	3.1
		40p	3.0	3.9	0.0	0.0	4.4	4.1	4.4	3.0	2.4	3.0
5	25	3p	3.0	5.6	3.0	10.0	4.0	3.8	4.0	4.5	4.0	4.2
		5p	3.0	5.0	3.0	9.2	4.2	4.1	4.2	4.6	3.9	4.1
		10p	3.0	4.9	3.0	8.4	4.7	4.3	4.7	4.7	4.0	4.1
		20p	3.0	4.9	3.0	7.2	4.9	4.8	4.9	4.8	4.0	4.1
		40p	3.0	5.0	3.0	5.5	5.0	4.9	5.0	4.8	4.0	4.0
5	50	3p	3.0	11.5	3.0	15.0	4.9	4.8	4.9	5.0	5.0	5.0
		5p	3.0	7.2	3.0	11.1	5.0	5.0	5.0	5.0	5.0	5.0
		10p	3.0	5.0	3.0	9.5	5.0	5.0	5.0	5.0	5.0	5.0
		20p	3.0	5.0	3.0	9.2	5.0	5.0	5.0	5.0	5.0	5.0
		40p	3.0	5.0	3.0	9.2	5.0	5.0	5.0	5.0	5.0	5.0
7	21	3p	3.0	5.8	3.0	9.4	3.3	2.8	3.3	3.2	3.2	4.1
		5p	3.0	5.6	3.0	8.4	4.3	3.7	4.3	3.4	3.0	4.0
		10p	3.0	5.6	3.0	7.1	5.3	4.6	5.3	3.5	2.9	4.0
		20p	3.0	5.8	3.0	5.7	6.1	5.2	6.1	3.5	2.8	4.0
		40p	3.0	5.9	3.0	4.3	6.3	5.8	6.3	3.5	2.8	4.0
7	35	3p	3.0	8.4	3.0	17.4	4.4	4.1	4.4	5.9	4.7	5.4
		5p	3.0	6.9	3.0	15.8	5.3	4.9	5.3	6.0	4.5	5.4
		10p	3.0	6.6	3.0	13.8	6.0	5.7	6.0	6.0	4.3	5.3
		20p	3.0	6.6	3.0	11.9	6.3	6.0	6.3	6.0	4.2	5.3
		40p	3.0	6.7	3.0	9.5	6.8	6.2	6.8	6.0	4.1	5.3
7	70	3p	3.0	19.4	3.0	33.9	5.7	5.5	5.7	7.0	6.4	6.9
		5p	3.0	13.0	3.0	30.1	6.3	6.1	6.3	7.0	6.2	6.8
		10p	3.0	7.6	3.0	24.1	6.9	6.8	6.9	7.0	6.1	6.8
		20p	3.0	7.0	3.0	20.5	7.0	6.9	7.0	7.0	6.0	6.8
		40p	3.0	7.0	3.0	18.4	7.0	7.0	7.0	7.0	6.0	6.8

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Table 6
Omega Squared for Mean Number of Factors Retained.

					Parallel Analysis					
Effect	CNG	SE_{scree}	MR	t	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
k	0.027	0.257	0.259	0.320	0.610	0.456	0.119	0.258	0.271	0.305
p/k	0.003	0.230	0.239	0.351	0.104	0.180	0.340	0.361	0.380	0.183
N	0.000	0.154	0.042	0.034	0.062	0.066	0.004	0.001	0.032	0.082
Communality	0.006	0.011	0.003	0.005	0.013	0.045	0.142	0.162	0.056	0.128
Phi	0.160	0.003	0.000	0.060	0.055	0.077	0.170	0.032	0.087	0.011
k X p/k	0.018	0.055	0.259	0.074	0.011	0.021	0.033	0.055	0.047	0.022
k X N	0.001	0.040	0.021	0.005	0.018	0.016	0.000	0.000	0.005	0.008
k X Comm	0.016	0.004	0.002	0.001	0.003	0.007	0.018	0.022	0.007	0.014
k X Phi	0.087	0.000	0.000	0.024	0.015	0.020	0.042	0.008	0.020	0.003
p/k X N	0.000	0.126	0.022	0.006	0.010	0.006	0.000	0.000	0.000	0.034
p/k X Comm	0.002	0.034	0.002	0.004	0.002	0.007	0.017	0.030	0.005	0.048
p/k X Phi	0.010	0.001	0.000	0.054	0.012	0.013	0.021	0.004	0.012	0.004
N X Comm	0.000	0.026	0.004	0.000	0.001	0.001	0.001	0.000	0.017	0.080
N X Phi	0.001	0.000	0.001	0.004	0.019	0.012	0.000	0.000	0.002	0.000
Comm X Phi	0.018	0.000	0.000	0.001	0.005	0.005	0.005	0.006	0.001	0.000
k X p/k X N	0.000	0.030	0.106	0.003	0.001	0.001	0.000	0.000	0.000	0.004
k X p/k X Comm	0.002	0.005	0.008	0.001	0.000	0.001	0.003	0.002	0.000	0.004
k X N X Comm	0.000	0.004	0.002	0.000	0.001	0.000	0.000	0.000	0.002	0.008
p/k X N X Comm	0.000	0.012	0.002	0.001	0.003	0.005	0.000	0.001	0.003	0.040
k X p/k X Phi	0.052	0.000	0.001	0.020	0.003	0.003	0.003	0.001	0.003	0.001
k X N X Phi	0.003	0.000	0.000	0.001	0.006	0.004	0.000	0.000	0.000	0.000
p/k X N X Phi	0.000	0.000	0.000	0.003	0.004	0.004	0.001	0.000	0.001	0.000
Comm X p/k X Phi	0.006	0.000	0.000	0.002	0.001	0.003	0.035	0.016	0.013	0.000
Comm X k X Phi	0.046	0.000	0.000	0.002	0.002	0.002	0.004	0.002	0.001	0.000
Comm X n X Phi	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
k X p/k X N X Comm	0.000	0.002	0.011	0.000	0.001	0.001	0.000	0.000	0.000	0.004
k X p/k X N X Phi	0.000	0.000	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000
Comm X k X p/k X Phi	0.008	0.000	0.000	0.003	0.001	0.001	0.004	0.004	0.002	0.001
Comm X p/k X n X Phi	0.000	0.000	0.000	0.000	0.003	0.005	0.001	0.000	0.002	0.000
Comm X k X n X Phi	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
k X p/k X N X Comm X phi	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.000

Table 7
Proportion of Samples Retaining K Factors, Phi = 0.00

			Low Communality										Wide Communality										High Communality									
k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	81	27	1	0	22	12	0	0	2	61	93	40	0	0	29	29	0	2	6	6	92	56	0	0	42	35	42	24	21	42
		5p	88	14	0	0	25	12	0	0	0	67	96	37	0	0	42	42	0	2	1	1	93	56	0	0	54	47	54	30	20	43
		10p	91	7	0	0	32	14	0	0	0	65	97	35	0	0	62	62	0	1	0	0	94	58	0	0	61	57	61	36	21	43
		20p	92	6	0	0	38	17	0	0	0	48	97	38	0	0	77	77	0	1	0	0	94	59	0	0	65	60	65	40	23	43
		40p	92	8	0	0	44	22	0	0	0	34	97	40	0	0	86	86	0	0	0	0	94	60	0	0	70	62	70	44	24	42
3	15	3p	90	21	92	39	49	46	0	7	63	0	88	33	89	26	83	83	17	71	86	86	87	89	85	22	96	93	96	95	94	99
		5p	92	57	93	41	63	63	0	6	23	2	89	75	87	19	89	89	29	80	80	80	89	100	85	18	100	99	100	98	94	100
		10p	94	85	92	37	81	85	0	6	7	21	90	95	83	9	94	94	43	87	79	79	91	100	69	12	100	100	100	99	96	100
		20p	96	93	63	21	92	97	0	6	2	81	91	96	48	3	98	98	54	89	79	79	91	100	9	1	100	100	100	99	97	100
		40p	96	98	6	2	98	100	0	7	1	100	91	97	7	0	99	99	60	89	79	79	92	100	0	0	100	100	100	100	98	100
3	30	3p	82	0	78	17	85	97	78	98	46	0	89	0	87	9	100	100	99	99	97	97	93	9	90	15	100	100	100	100	100	100
		5p	81	1	73	12	92	99	89	100	100	0	90	1	86	7	100	100	100	100	100	100	92	84	84	11	100	100	100	100	100	100
		10p	80	56	69	9	97	100	94	100	100	0	91	58	83	7	100	100	100	100	100	100	94	100	82	10	100	100	100	100	100	100
		20p	80	100	64	8	99	100	98	100	100	62	91	100	81	9	100	100	100	100	100	100	95	100	82	10	100	100	100	100	100	100
		40p	80	100	62	7	100	100	100	100	100	100	90	100	79	10	100	100	100	100	100	100	96	100	81	10	100	100	100	100	100	100
5	15	3p	5	30	3	19	14	4	0	0	0	43	4	39	3	20	14	14	0	0	0	0	11	73	9	32	36	22	36	14	4	24
		5p	4	17	3	28	20	5	0	0	0	55	5	27	2	32	22	22	0	0	0	0	11	75	10	53	59	40	59	20	3	24
		10p	3	7	1	32	33	6	0	0	0	66	7	15	1	25	40	40	0	0	0	0	11	78	1	7	81	62	81	23	2	25
		20p	2	6	0	7	51	12	0	0	0	53	9	12	0	4	60	60	0	0	0	0	12	82	0	0	93	77	93	24	1	25
		40p	1	8	0	0	63	21	0	0	0	40	12	14	0	0	70	70	0	0	0	0	13	86	0	0	98	88	98	25	0	25
5	25	3p	4	1	1	10	36	27	0	0	35	0	6	1	1	10	63	63	1	44	48	48	10	51	1	1	97	93	97	89	93	99
		5p	5	19	1	11	50	42	0	0	2	0	7	21	1	7	82	82	2	54	30	30	8	96	0	0	100	99	100	92	93	100
		10p	5	77	1	11	71	69	0	0	0	5	7	85	0	3	93	93	3	61	20	20	6	100	0	0	100	100	100	94	92	100
		20p	8	90	1	9	89	87	0	0	0	76	7	96	0	1	96	96	3	64	16	16	3	100	1	1	100	100	100	95	91	100
		40p	10	94	1	8	98	96	0	0	0	100	7	96	1	2	99	99	2	65	14	14	1	100	11	59	100	100	100	95	90	100
5	50	3p	4	0	0	1	95	97	37	95	23	0	2	0	0	0	100	100	100	100	98	98	5	0	0	0	100	100	100	98	100	100
		5p	4	0	0	0	98	100	45	99	100	0	1	0	0	0	100	100	100	100	100	100	3	3	0	0	100	100	100	99	100	100
		10p	3	1	0	0	99	100	47	100	100	0	0	0	0	0	100	100	100	100	100	100	1	99	0	0	100	100	100	99	100	100
		20p	2	78	0	0	100	100	46	100	100	38	0	57	0	0	100	100	100	100	100	100	0	100	0	0	100	100	100	99	100	100
		40p	1	100	0	0	100	100	44	100	100	100	0	100	0	0	100	100	100	100	100	100	0	100	0	0	100	100	100	99	100	100
7	21	3p	1	24	0	8	11	2	0	0	0	33	1	35	0	11	14	14	0	0	0	0	2	59	1	12	12	3	12	4	0	3
		5p	1	24	1	10	17	3	0	0	0	50	2	44	0	16	27	27	0	0	0	0	2	52	1	19	30	16	30	7	0	3
		10p	1	11	1	22	30	5	0	0	0	61	3	36	1	44	40	40	0	0	0	0	1	50	1	48	47	34	47	10	0	2
		20p	1	11	1	33	47	10	0	0	0	43	4	33	0	39	52	52	0	0	0	0	1	51	0	1	60	44	60	13	0	1
		40p	1	15	0	1	63	18	0	0	0	36	6	32	0	3	63	63	0	0	0	0	0	51	0	0	78	50	78	14	0	0
7	35	3p	2	0	0	8	33	16	0	0	21	0	2	0	0	6	61	61	0	43	43	43	5	5	0	0	93	82	93	84	82	99
		5p	3	2	0	10	45	32	0	0	0	0	3	1	0	3	80	80	0	53	25	25	3	79	0	0	100	99	100	86	77	100
		10p	5	48	0	8	61	64	0	0	0	1	2	53	0	1	89	89	0	56	17	17	2	100	0	0	100	100	100	87	71	100
		20p	9	82	0	4	76	79	0	0	0	71	2	93	0	0	92	92	0	57	16	16	1	100	0	0	100	100	100	85	66	100
		40p	11	81	0	1	88	80	0	0	0	82	2	91	0	0	97	97	0	57	16	16	0	100	0	9	100	100	100	85	63	100
7	70	3p	0	0	0	0	99	95	14	97	10	0	0	0	0	0	100	100	91	100	99	99	1	0	0	0	100	100	100	99	100	100
		5p	0	0	0	0	100	100	17	99	100	0	0	0	0	0	100	100	95	100	100	100	1	0	0	0	100	100	100	99	100	100
		10p	0	0	0	0	100	100	14	100	100	0	0	0	0	0	100	100	95	100	100	100	1	57	0	0	100	100	100	100	100	100
		20p	0	11	0	0	100	100	8	100	100	27	0	1	0	0	100	100	93	100	100	100	0	100	0	0	100	100	100	100	100	100
		40p	0	99	0	0	100	100	1	100	100	100	0	89	0	0	100	100	90	100	100	100	0	100	0	0	100	100	100	100	100	100

Table 8
Proportion of Samples Retaining K Factors, Phi = 0.30

			Low Communality										Wide Communality										High Communality									
k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	92	21	1	0	15	7	0	0	1	64	98	27	1	0	14	14	0	0	1	1	100	49	0	0	25	15	25	15	5	27
		5p	98	8	0	0	16	5	0	0	0	64	99	17	0	0	22	22	0	0	0	0	100	50	0	0	45	30	45	18	2	24
		10p	100	1	0	0	18	3	0	0	0	53	100	10	0	0	41	41	0	0	0	0	100	54	0	0	65	52	65	22	1	24
		20p	100	0	0	0	24	3	0	0	0	30	100	8	0	0	67	67	0	0	0	0	100	58	0	0	78	63	78	24	0	25
		40p	100	0	0	0	31	5	0	0	0	18	100	7	0	0	82	82	0	0	0	0	100	60	0	0	90	69	90	25	0	24
3	15	3p	100	27	100	33	33	20	0	2	33	1	100	38	100	33	51	51	1	33	43	43	100	91	100	19	78	68	78	90	72	91
		5p	100	47	100	37	43	25	0	1	3	6	100	75	100	31	73	73	1	38	24	24	100	99	100	13	93	89	93	96	72	92
		10p	100	48	99	38	63	43	0	0	0	40	100	92	98	23	93	93	1	39	17	17	100	100	85	8	100	97	100	99	71	92
		20p	100	51	77	27	80	61	0	0	0	86	100	96	67	12	98	98	2	39	15	15	100	100	15	1	100	100	100	100	70	91
		40p	100	56	15	5	93	72	0	0	0	86	100	99	12	3	99	99	2	36	16	16	100	100	0	0	100	100	100	100	70	90
3	30	3p	100	0	100	14	75	74	6	63	47	0	100	0	100	3	99	99	67	99	97	97	100	8	100	2	100	100	100	100	100	100
		5p	100	1	100	10	90	94	7	74	94	0	100	2	100	1	100	100	84	100	100	100	100	83	100	0	100	100	100	100	100	100
		10p	100	56	100	4	95	100	8	82	77	0	100	62	100	0	100	100	94	100	100	100	100	100	100	0	100	100	100	100	100	100
		20p	100	100	100	1	98	100	10	88	62	48	100	100	100	0	100	100	98	100	100	100	100	100	100	0	100	100	100	100	100	100
		40p	100	100	100	0	99	100	12	92	51	100	100	100	100	0	100	100	100	100	100	100	100	100	100	0	100	100	100	100	100	100
5	15	3p	0	21	0	16	4	0	0	0	0	57	0	27	0	17	3	3	0	0	0	0	0	26	0	24	1	0	1	1	0	0
		5p	0	5	0	29	4	0	0	0	0	58	0	10	0	37	5	5	0	0	0	0	0	21	0	61	6	1	6	0	0	0
		10p	0	0	0	40	5	0	0	0	0	35	0	3	0	41	14	14	0	0	0	0	0	23	0	18	28	7	28	0	0	0
		20p	0	0	0	18	9	0	0	0	0	8	0	2	0	12	27	27	0	0	0	0	0	28	0	0	53	23	53	0	0	0
		40p	0	0	0	1	16	0	0	0	0	1	0	3	0	2	39	39	0	0	0	0	0	32	0	0	73	37	73	0	0	0
5	25	3p	0	3	0	1	11	2	0	0	4	0	0	3	0	0	12	12	0	7	4	4	0	55	0	0	31	17	31	79	18	59
		5p	0	26	0	1	22	2	0	0	0	0	0	31	0	1	38	38	0	10	0	0	0	93	0	0	72	52	72	87	12	61
		10p	0	43	0	1	48	10	0	0	0	11	0	87	0	0	84	84	0	13	0	0	0	93	0	0	91	88	91	90	9	63
		20p	0	40	0	0	78	32	0	0	0	79	0	97	0	0	98	98	0	15	0	0	0	92	0	0	92	91	92	90	7	65
		40p	0	54	0	0	94	67	0	0	0	94	0	99	0	0	99	99	0	18	0	0	0	92	0	49	99	91	99	90	7	64
5	50	3p	0	0	0	0	49	23	0	28	37	0	0	0	0	0	95	95	16	99	98	98	0	0	0	0	96	94	96	100	100	100
		5p	0	0	0	0	82	62	0	37	64	0	0	0	0	0	100	100	32	100	100	100	0	3	0	0	100	99	100	100	99	100
		10p	0	2	0	0	95	90	0	44	24	0	0	0	0	0	100	100	47	100	99	99	0	99	0	0	100	100	100	100	99	100
		20p	0	83	0	0	100	96	0	48	10	49	0	53	0	0	100	100	51	100	98	98	0	100	0	0	100	100	100	100	98	100
		40p	0	100	0	0	100	100	0	50	7	100	0	100	0	0	100	100	51	100	98	98	0	100	0	0	100	100	100	100	98	100
7	21	3p	0	23	0	5	2	0	0	0	0	52	0	33	0	3	0	0	0	0	0	0	0	31	0	1	0	0	0	0	0	0
		5p	0	8	0	8	2	0	0	0	0	55	0	14	0	6	1	1	0	0	0	0	0	23	0	5	3	0	3	0	0	0
		10p	0	0	0	20	4	0	0	0	0	27	0	1	0	30	5	5	0	0	0	0	0	23	0	62	19	4	19	0	0	0
		20p	0	0	0	51	8	0	0	0	0	2	0	0	0	54	12	12	0	0	0	0	0	27	0	5	41	15	41	0	0	0
		40p	0	0	0	19	15	0	0	0	0	0	0	0	0	14	18	18	0	0	0	0	0	31	0	0	65	28	65	0	0	0
7	35	3p	0	0	0	0	3	0	0	0	0	0	0	0	0	0	4	4	0	2	0	0	0	11	0	0	19	7	19	68	8	48
		5p	0	8	0	0	7	0	0	0	0	0	0	2	0	0	27	27	0	4	0	0	0	84	0	0	59	44	59	75	5	52
		10p	0	39	0	0	24	0	0	0	0	18	0	50	0	0	74	74	0	6	0	0	0	100	0	0	84	70	84	80	4	55
		20p	0	25	0	0	47	6	0	0	0	54	0	96	0	0	95	95	0	8	0	0	0	100	0	0	100	88	100	84	4	57
		40p	0	28	0	0	68	20	0	0	0	42	0	97	0	0	99	99	0	9	0	0	0	100	0	2	100	100	100	84	3	58
7	70	3p	0	0	0	0	35	12	0	29	22	0	0	0	0	0	86	86	3	100	99	99	0	0	0	0	94	90	94	100	100	100
		5p	0	0	0	0	82	55	0	43	64	0	0	0	0	0	99	99	7	100	98	98	0	0	0	0	100	100	100	100	100	100
		10p	0	0	0	0	100	94	0	52	30	0	0	0	0	0	100	100	9	100	94	94	0	53	0	0	100	100	100	100	100	100
		20p	0	15	0	0	100	100	0	54	20	42	0	1	0	0	100	100	9	100	91	91	0	100	0	0	100	100	100	100	100	100
		40p	0	100	0	0	100	100	0	55	16	100	0	92	0	0	100	100	7	100	90	90	0	100	0	0	100	100	100	100	100	100

Table 9
Proportion of Samples Retaining K Factors, Phi = 0.50

			Low Communality										Wide Communality										High Communality									
k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	96	19	1	0	11	5	0	0	1	64	100	18	1	0	7	7	0	0	0	0	100	23	1	0	6	3	6	4	1	9
		5p	99	5	0	0	10	3	0	0	0	61	100	7	0	0	8	8	0	0	0	0	100	18	0	0	14	6	14	2	0	5
		10p	100	0	0	0	10	1	0	0	0	39	100	1	0	0	14	14	0	0	0	0	100	17	0	0	33	15	33	2	0	3
		20p	100	0	0	0	9	0	0	0	0	10	100	0	0	0	30	30	0	0	0	0	100	20	0	0	57	31	57	1	0	2
		40p	100	0	0	0	10	0	0	0	0	2	100	0	0	0	51	51	0	0	0	0	100	21	0	0	71	49	71	1	0	2
3	15	3p	100	26	100	21	18	7	0	0	17	2	100	40	100	24	19	19	0	13	14	14	100	85	100	20	26	15	26	67	19	58
		5p	100	33	100	24	22	6	0	0	0	7	100	59	100	27	37	37	0	15	2	2	100	95	100	14	60	40	60	78	11	56
		10p	100	17	99	26	37	11	0	0	0	39	100	63	98	25	66	66	0	17	0	0	100	98	91	8	94	81	94	84	4	55
		20p	100	14	88	21	63	25	0	0	0	80	100	65	76	17	82	82	0	20	0	0	100	100	24	1	100	98	100	85	2	57
		40p	100	19	34	6	85	45	0	0	0	73	100	66	23	3	95	95	0	23	0	0	100	100	0	0	100	100	100	85	1	58
3	30	3p	100	0	100	3	45	27	0	23	61	0	100	0	100	5	78	78	9	92	94	94	100	10	100	0	96	92	96	99	100	100
		5p	100	1	100	5	72	51	0	25	49	0	100	1	100	4	94	94	17	97	88	88	100	84	100	0	100	100	100	100	100	100
		10p	100	61	100	5	95	88	0	26	15	0	100	59	100	3	100	100	26	99	81	81	100	100	100	0	100	100	100	100	100	100
		20p	100	100	100	2	98	99	0	27	10	69	100	100	100	1	100	100	34	100	77	77	100	100	100	0	100	100	100	100	100	100
		40p	100	100	100	0	99	100	0	25	9	100	100	100	100	0	100	100	39	100	74	74	100	100	100	0	100	100	100	100	100	100
5	15	3p	0	18	0	16	2	0	0	0	0	57	0	17	0	16	0	0	0	0	0	0	0	10	0	21	1	0	1	0	0	0
		5p	0	4	0	29	1	0	0	0	0	52	0	4	0	38	1	1	0	0	0	0	0	7	0	55	3	1	3	0	0	0
		10p	0	0	0	39	2	0	0	0	0	18	0	1	0	45	4	4	0	0	0	0	0	8	0	25	10	5	10	0	0	0
		20p	0	0	0	19	2	0	0	0	0	1	0	1	0	14	15	15	0	0	0	0	0	10	0	1	13	10	13	0	0	0
		40p	0	0	0	2	5	0	0	0	0	0	0	0	0	2	32	32	0	0	0	0	0	10	0	0	28	10	28	0	0	0
5	25	3p	0	6	0	0	1	0	0	0	0	0	0	6	0	0	1	1	0	1	0	0	0	57	0	0	1	0	1	37	0	12
		5p	0	29	0	1	1	0	0	0	0	5	0	39	0	0	6	6	0	1	0	0	0	88	0	0	14	3	14	48	0	8
		10p	0	7	0	1	5	0	0	0	0	45	0	65	0	0	38	38	0	0	0	0	0	91	0	0	63	37	63	57	0	5
		20p	0	1	0	1	21	0	0	0	0	27	0	67	0	0	79	79	0	0	0	0	0	97	0	0	94	69	94	60	0	4
		40p	0	2	0	2	47	4	0	0	0	8	0	73	0	0	90	90	0	0	0	0	0	100	0	28	100	91	100	61	0	4
5	50	3p	0	0	0	0	4	0	0	1	52	0	0	0	0	0	21	21	0	78	78	78	0	0	0	0	60	40	60	100	93	100
		5p	0	0	0	0	23	4	0	1	5	0	0	0	0	0	66	66	0	89	46	46	0	5	0	0	96	90	96	100	90	100
		10p	0	2	0	0	77	38	0	1	0	0	0	1	0	0	99	99	0	96	27	27	0	99	0	0	100	100	100	100	89	100
		20p	0	81	0	0	93	83	0	1	0	50	0	68	0	0	100	100	0	98	21	21	0	100	0	0	100	100	100	100	89	100
		40p	0	95	0	0	98	92	0	0	0	95	0	100	0	0	100	100	0	99	16	16	0	100	0	0	100	100	100	100	90	100
7	21	3p	0	20	0	5	0	0	0	0	0	44	0	23	0	3	0	0	0	0	0	0	0	9	0	1	0	0	0	0	0	0
		5p	0	3	0	8	0	0	0	0	0	23	0	5	0	6	0	0	0	0	0	0	0	2	0	4	0	0	0	0	0	0
		10p	0	0	0	18	0	0	0	0	0	1	0	0	0	25	1	1	0	0	0	0	0	1	0	58	1	0	1	0	0	0
		20p	0	0	0	44	0	0	0	0	0	0	0	0	0	61	5	5	0	0	0	0	0	2	0	13	13	0	13	0	0	0
		40p	0	0	0	29	1	0	0	0	0	0	0	0	0	21	15	15	0	0	0	0	0	4	0	0	33	1	33	0	0	0
7	35	3p	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	0	0	0	0	0	18	0	2
		5p	0	14	0	0	0	0	0	0	0	0	0	6	0	0	4	4	0	1	0	0	0	79	0	0	3	0	3	23	0	1
		10p	0	19	0	0	3	0	0	0	0	32	0	48	0	0	29	29	0	1	0	0	0	82	0	0	38	11	38	29	0	0
		20p	0	4	0	0	23	0	0	0	0	28	0	52	0	0	53	53	0	2	0	0	0	82	0	0	78	51	78	29	0	0
		40p	0	4	0	0	52	2	0	0	0	5	0	50	0	0	73	73	0	2	0	0	0	83	0	0	91	77	91	29	0	0
7	70	3p	0	0	0	0	0	0	0	0	47	0	0	0	0	0	10	10	0	61	59	59	0	0	0	0	39	24	39	100	84	99
		5p	0	0	0	0	6	0	0	0	1	0	0	0	0	0	41	41	0	70	33	33	0	0	0	0	85	74	85	100	78	99
		10p	0	0	0	0	70	26	0	0	0	0	0	0	0	0	87	87	0	76	19	19	0	54	0	0	100	99	100	100	74	99
		20p	0	17	0	0	98	78	0	0	0	43	0	2	0	0	96	96	0	80	13	13	0	100	0	0	100	100	100	100	70	99
		40p	0	100	0	0	100	96	0	0	0	100	0	93	0	0	100	100	0	81	10	10	0	100	0	0	100	100	100	100	70	99

Table 10
Proportion of Samples Retaining K Factors, Phi = Mixed

			Low Communality										Wide Communality										High Communality									
k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	93	22	1	0	15	7	0	0	1	64	99	27	1	0	13	13	0	1	1	1	99	29	1	0	10	5	10	10	1	10
		5p	98	7	0	0	14	4	0	0	0	67	100	13	0	0	18	18	0	0	0	0	100	25	0	0	19	9	19	9	0	6
		10p	100	1	0	0	16	2	0	0	0	55	100	6	0	0	33	33	0	0	0	0	100	24	0	0	39	21	39	8	0	3
		20p	100	0	0	0	18	2	0	0	0	24	100	3	0	0	52	52	0	0	0	0	100	25	0	0	62	35	62	6	0	1
		40p	100	0	0	0	23	1	0	0	0	7	100	2	0	0	69	69	0	0	0	0	100	26	0	0	77	52	77	5	0	0
3	15	3p	95	23	96	38	43	36	0	5	49	0	100	41	100	50	25	25	0	15	18	18	100	91	100	37	53	38	53	84	44	80
		5p	97	52	97	42	54	46	0	5	13	3	100	64	100	58	41	41	0	15	3	3	100	99	100	31	85	70	85	92	34	80
		10p	99	68	97	37	72	67	0	5	4	27	100	67	98	58	71	71	0	15	0	0	100	100	87	21	100	97	100	97	26	82
		20p	100	72	71	22	87	79	0	6	1	85	100	69	64	35	93	93	0	14	0	0	100	100	19	5	100	100	100	98	22	82
		40p	100	76	10	3	97	87	0	7	0	97	100	77	9	4	99	99	0	13	0	0	100	100	0	0	100	100	100	99	21	82
3	30	3p	100	0	100	55	65	55	2	46	51	0	100	0	100	53	88	88	29	93	95	95	100	10	100	9	100	99	100	100	100	100
		5p	100	1	100	58	84	76	2	51	75	0	100	1	100	50	96	96	40	96	92	92	100	84	100	6	100	100	100	100	100	100
		10p	100	57	100	52	96	96	1	54	43	0	100	55	100	47	100	100	48	97	89	89	100	100	100	4	100	100	100	100	100	100
		20p	100	100	100	46	98	100	0	56	26	56	100	99	100	44	100	100	55	99	89	89	100	100	100	2	100	100	100	100	100	100
		40p	100	100	100	43	99	100	0	57	21	100	100	100	100	41	100	100	60	100	89	89	100	100	100	1	100	100	100	100	100	100
5	15	3p	0	21	0	15	3	0	0	0	0	56	0	24	0	14	2	2	0	0	0	0	0	15	0	23	0	0	0	0	0	0
		5p	0	5	0	28	3	0	0	0	0	53	0	8	0	27	3	3	0	0	0	0	0	8	0	49	2	0	2	0	0	0
		10p	0	0	0	36	4	0	0	0	0	25	0	1	0	32	7	7	0	0	0	0	0	8	0	19	10	2	10	0	0	0
		20p	0	0	0	14	5	0	0	0	0	4	0	0	0	10	21	21	0	0	0	0	0	10	0	0	23	8	23	0	0	0
		40p	0	0	0	1	5	0	0	0	0	0	0	0	0	1	41	41	0	0	0	0	0	13	0	0	39	17	39	0	0	0
5	25	3p	0	3	0	1	9	1	0	0	3	0	0	1	0	1	32	32	0	16	15	15	0	51	0	1	4	1	4	53	1	21
		5p	0	29	0	2	13	1	0	0	0	2	0	23	0	1	62	62	0	24	4	4	0	91	0	1	18	6	18	65	0	13
		10p	0	19	0	4	20	1	0	0	0	34	0	82	0	0	87	87	0	32	1	1	0	94	0	0	70	31	70	74	0	6
		20p	0	3	0	6	30	1	0	0	0	43	0	93	0	0	96	96	0	36	1	1	0	94	0	0	93	79	93	81	0	5
		40p	0	0	0	5	50	1	0	0	0	9	0	94	0	0	99	99	0	37	0	0	0	96	0	46	100	91	100	86	0	4
5	50	3p	0	0	0	0	8	1	0	3	46	0	0	0	0	0	74	74	2	96	97	97	0	0	0	0	87	78	87	100	99	100
		5p	0	0	0	0	34	8	0	2	10	0	0	0	0	0	97	97	5	100	91	91	0	3	0	0	100	98	100	100	98	100
		10p	0	2	0	0	85	50	0	2	0	0	0	0	0	0	100	100	6	100	84	84	0	99	0	0	100	100	100	100	98	100
		20p	0	83	0	0	99	88	0	2	0	42	0	64	0	0	100	100	6	100	82	82	0	100	0	0	100	100	100	100	97	100
		40p	0	100	0	0	100	100	0	2	0	100	0	100	0	0	100	100	4	100	81	81	0	100	0	0	100	100	100	100	97	100
7	21	3p	0	23	0	4	1	0	0	0	0	53	0	28	0	3	0	0	0	0	0	0	0	16	0	1	0	0	0	0	0	0
		5p	0	6	0	8	1	0	0	0	0	42	0	8	0	7	0	0	0	0	0	0	0	7	0	5	0	0	0	0	0	0
		10p	0	0	0	19	1	0	0	0	0	7	0	0	0	29	1	1	0	0	0	0	0	7	0	62	4	0	4	0	0	0
		20p	0	0	0	41	1	0	0	0	0	0	0	0	0	49	2	2	0	0	0	0	0	10	0	9	27	1	27	0	0	0
		40p	0	0	0	10	1	0	0	0	0	0	0	0	0	9	2	2	0	0	0	0	0	14	0	0	41	11	41	0	0	0
7	35	3p	0	0	0	0	3	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	17	0	0	0	0	0	11	0	0
		5p	0	8	0	0	5	0	0	0	0	0	0	5	0	0	5	5	0	1	0	0	0	66	0	0	0	0	0	11	0	0
		10p	0	32	0	0	11	0	0	0	0	15	0	53	0	0	36	36	0	1	0	0	0	55	0	0	2	0	2	11	0	0
		20p	0	2	0	0	14	0	0	0	0	30	0	71	0	0	69	69	0	1	0	0	0	58	0	0	32	2	32	10	0	0
		40p	0	0	0	0	16	0	0	0	0	0	0	70	0	0	88	88	0	2	0	0	0	66	0	0	77	20	77	11	0	0
7	70	3p	0	0	0	0	1	0	0	2	39	0	0	0	0	0	0	0	0	50	37	37	0	0	0	0	6	2	6	98	38	86
		5p	0	0	0	0	15	2	0	3	5	0	0	0	0	0	6	6	0	64	1	1	0	0	0	0	34	16	34	99	21	83
		10p	0	0	0	0	71	33	0	4	0	0	0	0	0	0	73	73	0	78	0	0	0	51	0	0	92	82	92	100	13	80
		20p	0	17	0	0	99	77	0	6	0	48	0	2	0	0	90	90	0	85	0	0	0	100	0	0	100	92	100	100	7	80
		40p	0	100	0	0	100	97	0	8	0	100	0	96	0	0	92	92	0	88	0	0	0	100	0	0	100	100	100	100	4	80

Table 11
Omega Squared for Proportion of Samples Retaining K Factors.

					Parallel Analysis					
Effect	CNG	SE_{scree}	MR	t	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
k	0.975	0.043	0.476	0.011	0.043	0.043	0.036	0.014	0.028	0.033
p/k	0.000	0.158	0.126	0.041	0.366	0.403	0.255	0.473	0.467	0.192
N	0.000	0.154	0.011	0.022	0.133	0.082	0.003	0.001	0.004	0.033
Communality	0.000	0.118	0.000	0.005	0.044	0.049	0.072	0.156	0.062	0.047
Phi	0.000	0.023	0.001	0.011	0.089	0.094	0.130	0.044	0.082	0.044
k X p/k	0.000	0.012	0.259	0.192	0.003	0.004	0.036	0.004	0.013	0.011
k X N	0.000	0.006	0.022	0.035	0.002	0.001	0.001	0.000	0.000	0.000
k X Comm	0.000	0.000	0.001	0.015	0.001	0.001	0.005	0.002	0.002	0.001
k X Phi	0.003	0.001	0.004	0.031	0.013	0.008	0.002	0.002	0.005	0.002
p/k X N	0.000	0.174	0.020	0.032	0.014	0.014	0.001	0.000	0.003	0.090
p/k X Comm	0.000	0.020	0.001	0.002	0.004	0.005	0.065	0.076	0.039	0.146
p/k X Phi	0.000	0.014	0.002	0.015	0.007	0.015	0.109	0.011	0.040	0.029
N X Comm	0.000	0.013	0.001	0.010	0.000	0.000	0.001	0.000	0.001	0.034
N X Phi	0.000	0.003	0.000	0.002	0.007	0.004	0.000	0.000	0.002	0.004
Comm X Phi	0.000	0.001	0.000	0.004	0.001	0.002	0.018	0.002	0.004	0.008
k X p/k X N	0.000	0.021	0.041	0.110	0.007	0.007	0.000	0.000	0.001	0.003
k X p/k X Comm	0.000	0.002	0.003	0.012	0.003	0.004	0.003	0.012	0.005	0.003
k X N X Comm	0.000	0.009	0.001	0.021	0.000	0.000	0.000	0.000	0.001	0.001
p/k X N X Comm	0.000	0.027	0.001	0.027	0.014	0.011	0.000	0.001	0.002	0.093
k X p/k X Phi	0.000	0.001	0.002	0.022	0.005	0.007	0.007	0.003	0.023	0.008
k X N X Phi	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000
p/k X N X Phi	0.000	0.006	0.000	0.001	0.020	0.018	0.002	0.000	0.011	0.005
Comm X p/k X Phi	0.000	0.015	0.000	0.009	0.004	0.009	0.035	0.057	0.042	0.013
Comm X k X Phi	0.000	0.002	0.000	0.010	0.003	0.004	0.011	0.002	0.004	0.003
Comm X n X Phi	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.002	0.002
k X p/k X N X Comm	0.000	0.028	0.003	0.037	0.003	0.003	0.000	0.000	0.000	0.002
k X p/k X N X Phi	0.000	0.000	0.000	0.003	0.009	0.006	0.000	0.000	0.001	0.001
Comm X k X p/k X Phi	0.000	0.004	0.000	0.014	0.004	0.006	0.049	0.009	0.016	0.008
Comm X p/k X n X Phi	0.000	0.004	0.000	0.001	0.009	0.013	0.001	0.000	0.009	0.008
Comm X k X n X Phi	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
k X p/k X N X Comm X phi	0.000	0.000	0.000	0.000	0.003	0.005	0.001	0.000	0.000	0.003

Table 12
Proportion of Samples Agreeing with Rule Applied to Population R-Matrix, Phi = 0.00

			Low Communality										Wide Communality										High Communality									
k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	0	40	99	99	27	32	42	87	35	22	0	52	99	99	31	31	62	67	49	49	0	87	100	100	52	64	52	62	75	84
		5p	0	57	100	100	35	43	48	90	67	29	0	62	100	100	47	47	71	73	64	64	0	93	100	100	64	77	64	71	79	88
		10p	0	74	100	100	45	57	57	94	84	47	0	72	100	100	68	68	78	81	79	79	0	97	100	100	71	87	71	79	85	92
		20p	0	84	100	100	56	70	66	96	90	74	0	80	100	100	85	85	84	87	89	89	0	99	100	100	75	90	75	85	89	95
		40p	0	91	100	100	65	82	73	98	93	91	0	88	100	100	95	95	89	92	95	95	0	100	100	100	80	92	80	90	93	97
3	15	3p	0	21	0	0	49	46	43	53	25	0	0	33	0	0	83	83	33	79	85	85	0	89	0	0	96	93	96	95	94	99
		5p	0	57	0	0	63	63	47	60	54	2	0	75	0	0	89	89	46	90	87	87	0	100	0	0	100	99	100	98	94	100
		10p	0	85	1	1	81	85	50	71	68	21	0	95	4	4	94	94	62	97	90	90	0	100	19	19	100	100	100	99	96	100
		20p	0	93	32	32	92	97	55	80	77	81	0	96	46	46	98	98	74	99	93	93	0	100	90	90	100	100	100	99	97	100
		40p	0	98	94	94	98	100	62	87	83	100	0	97	93	93	99	99	80	99	96	96	0	100	100	100	100	100	100	100	98	100
3	30	3p	0	0	0	0	85	97	78	98	46	0	0	0	0	0	100	100	99	99	97	97	0	9	0	0	100	100	100	100	100	100
		5p	0	1	0	0	92	99	89	100	100	0	0	1	0	0	100	100	100	100	100	100	0	84	0	0	100	100	100	100	100	100
		10p	0	56	0	0	97	100	94	100	100	0	0	58	0	0	100	100	100	100	100	100	0	100	0	0	100	100	100	100	100	100
		20p	0	100	0	0	99	100	98	100	100	62	0	100	0	0	100	100	100	100	100	100	0	100	0	0	100	100	100	100	100	100
		40p	0	100	0	0	100	100	100	100	100	100	0	100	0	0	100	100	100	100	100	100	0	100	0	0	100	100	100	100	100	100
5	15	3p	0	28	0	0	15	8	64	69	2	14	0	36	0	0	22	22	48	46	39	39	0	75	0	0	36	22	36	35	72	77
		5p	0	33	0	0	23	11	73	73	13	21	0	54	0	0	36	36	58	51	63	63	0	81	0	0	59	40	59	46	78	83
		10p	0	31	1	1	39	17	82	79	43	41	0	66	2	2	60	60	67	58	82	82	0	86	31	31	81	62	81	53	85	89
		20p	0	34	22	22	62	34	88	85	68	66	0	74	32	32	83	83	77	65	91	91	0	91	95	95	93	77	93	60	89	93
		40p	0	44	85	85	81	57	91	90	84	82	0	82	83	83	95	95	85	72	94	94	0	96	100	100	98	88	98	65	93	94
5	25	3p	0	1	0	0	36	27	49	40	2	0	0	1	0	0	63	63	55	59	59	59	0	51	0	0	97	93	97	89	93	99
		5p	0	19	0	0	50	42	56	48	29	0	0	20	0	0	82	82	70	73	76	76	0	96	0	0	100	99	100	92	95	100
		10p	0	77	0	0	71	69	64	57	60	5	0	86	0	0	93	93	81	83	85	85	0	100	0	0	100	100	100	94	98	100
		20p	0	90	0	0	89	87	74	63	73	76	0	99	0	0	96	96	86	89	90	90	0	100	0	0	100	100	100	95	99	100
		40p	0	94	0	0	98	96	81	68	79	100	0	100	0	0	99	99	91	91	94	94	0	100	1	1	100	100	100	95	100	100
5	50	3p	0	0	0	0	95	97	72	95	23	0	0	0	0	0	100	100	100	100	98	98	0	0	0	0	100	100	100	98	100	100
		5p	0	0	0	0	98	100	80	99	100	0	0	0	0	0	100	100	100	100	100	100	0	3	0	0	100	100	100	99	100	100
		10p	0	1	0	0	99	100	87	100	100	0	0	0	0	0	100	100	100	100	100	100	0	99	0	0	100	100	100	99	100	100
		20p	0	78	0	0	100	100	92	100	100	38	0	57	0	0	100	100	100	100	100	100	0	100	0	0	100	100	100	99	100	100
		40p	0	100	0	0	100	100	95	100	100	100	0	100	0	0	100	100	100	100	100	100	0	100	0	0	100	100	100	99	100	100
7	21	3p	0	17	70	17	14	6	52	56	0	8	0	22	81	11	17	17	40	30	30	30	0	68	93	16	12	19	12	27	65	75
		5p	0	31	73	18	22	9	57	64	8	14	0	52	82	8	30	30	53	37	48	48	0	85	96	14	30	38	30	37	71	80
		10p	0	31	76	18	39	17	61	75	33	34	0	78	83	6	45	45	66	47	67	67	0	93	98	17	47	62	47	50	77	88
		20p	0	34	78	21	63	29	65	83	58	56	0	86	82	10	58	58	74	57	80	80	0	96	99	52	60	74	60	63	84	94
		40p	0	45	80	34	86	44	70	89	74	68	0	87	81	24	71	71	80	69	88	88	0	99	100	85	78	80	78	76	89	96
7	35	3p	0	0	92	11	33	22	48	35	8	0	0	0	95	13	61	61	59	59	45	45	0	5	97	3	93	82	93	76	73	99
		5p	0	1	95	8	45	42	55	45	39	0	0	1	96	11	80	80	65	69	56	56	0	79	99	1	100	99	100	78	79	100
		10p	0	48	98	5	61	81	64	57	63	0	0	48	98	9	89	89	69	75	72	72	0	100	100	0	100	100	100	80	87	100
		20p	0	90	99	3	76	99	72	62	78	64	0	95	100	7	92	92	75	80	82	82	0	100	100	0	100	100	100	80	92	100
		40p	0	91	100	3	88	100	80	67	84	98	0	99	100	6	97	97	82	84	89	89	0	100	99	0	100	100	100	81	96	100
7	70	3p	0	0	100	2	99	95	57	97	10	0	0	0	99	1	100	100	94	100	99	99	0	0	100	2	100	100	100	99	100	100
		5p	0	0	100	1	100	100	55	99	100	0	0	0	99	1	100	100	94	100	100	100	0	0	100	1	100	100	100	99	100	100
		10p	0	0	100	0	100	100	58	100	100	0	0	0	100	0	100	100	95	100	100	100	0	57	100	0	100	100	100	100	100	100
		20p	0	11	100	0	100	100	67	100	100	27	0	1	100	0	100	100	97	100	100	100	0	100	100	0	100	100	100	100	100	100
		40p	0	99	100	0	100	100	77	100	100	100	0	89	100	0	100	100	100	100	100	100	0	100	100	0	100	100	100	100	100	100

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Table 13
Proportion of Samples Agreeing with Rule Applied to Population R-Matrix, Phi = 0.30

Low Communality

k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	0	37	99	99	25	27	66	92	51	17
		5p	0	54	100	100	30	32	77	96	88	25
		10p	0	68	100	100	39	40	89	99	99	48
		20p	0	74	100	100	52	54	97	100	100	72
		40p	0	81	100	100	65	71	100	100	100	86
3	15	3p	0	21	0	0	33	23	88	69	11	1
		5p	0	48	0	0	43	29	90	76	39	4
		10p	0	66	1	1	63	50	91	85	71	26
		20p	0	76	23	23	80	70	91	92	85	70
		40p	0	84	85	85	93	81	91	96	90	83
3	30	3p	0	0	0	0	75	74	44	63	19	0
		5p	0	1	0	0	90	94	51	74	46	0
		10p	0	56	0	0	95	100	59	82	62	0
		20p	0	100	0	0	98	100	67	88	78	48
		40p	0	100	0	0	99	100	74	92	88	100
5	15	3p	0	18	0	0	6	8	97	88	8	1
		5p	0	34	0	0	6	8	100	95	64	2
		10p	0	39	0	0	10	10	100	99	98	11
		20p	0	39	9	9	17	20	100	100	100	40
		40p	0	43	59	59	28	38	100	100	100	66
5	25	3p	0	3	0	0	11	2	96	58	0	0
		5p	0	26	0	0	22	2	98	66	8	0
		10p	0	43	0	0	48	10	99	76	41	9
		20p	0	40	0	0	78	32	100	84	69	65
		40p	0	54	0	0	94	67	100	92	81	84
5	50	3p	0	0	0	0	49	23	29	54	4	0
		5p	0	0	0	0	82	62	36	68	31	0
		10p	0	2	0	0	95	90	43	79	58	0
		20p	0	83	0	0	100	96	52	88	75	49
		40p	0	100	0	0	100	100	61	93	85	100
7	21	3p	0	11	60	3	3	2	99	81	0	1
		5p	0	29	60	3	5	2	100	91	23	2
		10p	0	23	60	3	10	5	100	97	87	10
		20p	0	17	60	3	23	16	100	99	99	33
		40p	0	21	60	6	42	38	100	100	100	55
7	35	3p	0	0	100	0	3	0	70	45	0	0
		5p	0	6	100	0	9	1	72	54	3	0
		10p	0	43	100	0	30	10	74	64	31	2
		20p	0	47	100	0	54	31	75	75	65	42
		40p	0	49	100	0	75	51	77	82	83	70
7	70	3p	0	0	100	0	35	12	51	50	4	0
		5p	0	0	100	0	82	55	59	69	24	0
		10p	0	0	100	0	100	94	69	82	46	0
		20p	0	15	100	0	100	100	80	87	65	42
		40p	0	100	100	0	100	100	89	90	75	100

Wide Communality

k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	0	48	99	99	14	14	87	75	49	49
		5p	0	56	100	100	22	22	89	81	75	75
		10p	0	63	100	100	41	41	90	89	90	90
		20p	0	70	100	100	67	67	90	93	95	95
		40p	0	79	100	100	82	82	90	95	96	96
3	15	3p	0	38	0	0	51	51	43	66	54	54
		5p	0	75	0	0	73	73	51	75	69	69
		10p	0	92	2	2	93	93	63	83	78	78
		20p	0	96	33	33	98	98	75	88	84	84
		40p	0	99	88	88	99	99	82	93	88	88
3	30	3p	0	0	0	0	99	99	67	99	97	97
		5p	0	2	0	0	100	100	84	100	100	100
		10p	0	62	0	0	100	100	94	100	100	100
		20p	0	100	0	0	100	100	98	100	100	100
		40p	0	100	0	0	100	100	100	100	100	100
5	15	3p	0	31	0	0	6	6	80	51	11	11
		5p	0	44	0	0	12	12	81	57	29	29
		10p	0	52	1	1	26	26	82	65	57	57
		20p	0	58	21	21	42	42	83	73	76	76
		40p	0	64	66	66	57	57	84	82	86	86
5	25	3p	0	3	0	0	12	12	35	39	22	22
		5p	0	31	0	0	38	38	45	47	43	43
		10p	0	87	0	0	84	84	55	58	61	61
		20p	0	97	0	0	98	98	66	67	73	73
		40p	0	99	0	0	99	99	76	73	83	83
5	50	3p	0	0	0	0	95	95	51	99	98	98
		5p	0	0	0	0	100	100	63	100	100	100
		10p	0	0	0	0	100	100	76	100	99	99
		20p	0	53	0	0	100	100	84	100	98	98
		40p	0	100	0	0	100	100	90	100	98	98
7	21	3p	0	25	70	4	2	2	59	45	9	9
		5p	0	43	70	3	6	6	66	53	32	32
		10p	0	49	70	3	28	28	77	66	67	67
		20p	0	56	70	2	55	55	88	73	83	83
		40p	0	66	70	10	68	68	97	82	91	91
7	35	3p	0	0	100	0	4	4	40	33	9	9
		5p	0	2	100	0	27	27	53	43	29	29
		10p	0	50	100	0	74	74	67	54	56	56
		20p	0	96	100	0	95	95	74	63	71	71
		40p	0	97	100	0	99	99	81	70	82	82
7	70	3p	0	0	100	0	86	86	62	100	89	89
		5p	0	0	100	0	99	99	51	100	92	92
		10p	0	0	100	0	100	100	42	100	96	96
		20p	0	1	100	0	100	100	46	100	99	99
		40p	0	92	100	0	100	100	62	100	100	100

High Communality

k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	0	71	100	100	25	23	25	56	59	72
		5p	0	80	100	100	45	40	45	63	65	78
		10p	0	90	100	100	65	62	65	71	70	82
		20p	0	95	100	100	78	73	78	77	75	87
		40p	0	99	100	100	90	79	90	82	80	91
3	15	3p	0	91	0	0	78	68	78	90	75	92
		5p	0	99	0	0	93	89	93	96	79	96
		10p	0	100	15	15	100	97	100	99	81	98
		20p	0	100	85	85	100	100	100	100	85	99
		40p	0	100	100	100	100	100	100	100	88	100
3	30	3p	0	8	0	0	100	100	100	100	100	100
		5p	0	83	0	0	100	100	100	100	100	100
		10p	0	100	0	0	100	100	100	100	100	100
		20p	0	100	0	0	100	100	100	100	100	100
		40p	0	100	0	0	100	100	100	100	100	100
5	15	3p	0	59	0	0	5	3	5	34	40	63
		5p	0	66	0	0	15	11	15	44	50	71
		10p	0	76	14	14	38	28	38	54	61	82
		20p	0	85	83	83	63	52	63	62	71	90
		40p	0	90	100	100	83	67	83	69	81	96
5	25	3p	0	55	0	0	31	17	31	87	57	74
		5p	0	93	0	0	72	52	72	96	68	80
		10p	0	93	0	0	91	88	91	99	80	87
		20p	0	92	0	0	92	91	92	99	89	92
		40p	0	92	0	0	99	91	99	99	93	97
5	50	3p	0	0	0	0	96	94	96	100	100	100
		5p	0	3	0	0	100	99	100	100	99	100
		10p	0	99	0	0	100	100	100	100	99	100
		20p	0	100	0	0	100	100	100	100	98	100
		40p	0	100	0	0	100	100	100	100	98	100
7	21	3p	0	60	90	0	5	2	5	31	42	55
		5p	0	67	90	0	12	11	12	41	53	63
		10p	0	75	90	1	31	28	31	52	65	74
		20p	0	85	90	13	60	53	60	63	76	82
		40p	0	92	91	49	85	80	85	72	85	89
7	35	3p	0	11	100	0	19	7	19	73	58	73
		5p	0	84	100	0	59	44	59	81	70	82
		10p	0	100	100	0	84	70	84	86	78	90
		20p	0	100	100	0	100	88	100	90	82	94
		40p	0	100	100	0	100	100	100	93	85	97
7	70	3p	0	0	100	0	94	90	94	100	100	100
		5p	0	0	100	0	100	100	100	100	100	100
		10p	0	53	100	0	100	100	100	100	100	100
		20p	0	100	100	0	100	100	100	100	100	100
		40p	0	100	100	0	100	100	100	100	100	100

Table 14
Proportion of Samples Agreeing with Rule Applied to Population R-Matrix, Phi = 0.50

			Low Communality										Wide Communality										High Communality									
k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	0	38	99	99	25	31	77	94	62	11	0	46	99	99	9	9	99	77	68	68	0	61	99	99	6	13	6	64	62	66
		5p	0	62	100	100	29	32	89	98	95	20	0	58	100	100	11	11	100	81	92	92	0	67	100	100	14	21	14	72	76	73
		10p	0	77	100	100	36	36	98	100	100	40	0	68	100	100	19	19	100	85	98	98	0	75	100	100	34	40	34	82	87	81
		20p	0	81	100	100	48	46	100	100	100	67	0	74	100	100	38	38	100	89	100	100	0	81	100	100	57	59	57	89	94	87
		40p	0	83	100	100	62	61	100	100	100	82	0	80	100	100	59	59	100	93	100	100	0	86	100	100	71	77	71	92	96	91
3	15	3p	0	19	0	0	18	10	100	89	16	1	0	35	0	0	19	19	83	51	30	30	0	85	0	0	26	15	26	71	56	67
		5p	0	41	0	0	22	9	100	95	77	5	0	65	0	0	37	37	84	59	54	54	0	95	0	0	60	40	60	83	64	73
		10p	0	41	1	1	37	15	100	99	99	25	0	79	2	2	66	66	86	69	74	74	0	98	9	9	94	81	94	90	72	80
		20p	0	40	12	12	63	31	100	100	100	60	0	84	24	24	82	82	89	78	82	82	0	100	76	76	100	98	100	93	79	86
		40p	0	49	66	66	85	54	100	100	100	77	0	86	77	77	95	95	91	86	88	88	0	100	100	100	100	100	100	95	82	90
3	30	3p	0	0	0	0	45	27	75	61	7	0	0	0	0	0	78	78	38	92	72	72	0	10	0	0	96	92	96	99	100	100
		5p	0	1	0	0	72	51	77	67	31	0	0	1	0	0	94	94	53	97	81	81	0	84	0	0	100	100	100	100	100	100
		10p	0	61	0	0	95	88	77	77	53	0	0	59	0	0	100	100	67	99	87	87	0	100	0	0	100	100	100	100	100	100
		20p	0	100	0	0	98	99	80	84	68	69	0	100	0	0	100	100	77	100	92	92	0	100	0	0	100	100	100	100	100	100
		40p	0	100	0	0	99	100	84	91	78	100	0	100	0	0	100	100	86	100	95	95	0	100	0	0	100	100	100	100	100	100
5	15	3p	0	9	0	0	6	10	99	91	15	0	0	28	0	0	1	1	100	63	28	28	0	58	0	0	4	2	4	47	39	61
		5p	0	32	0	0	5	8	100	97	83	1	0	40	0	0	1	1	100	68	72	72	0	67	0	0	11	7	11	55	53	68
		10p	0	56	0	0	7	8	100	99	99	4	0	51	1	1	5	5	100	74	97	97	0	79	8	8	20	22	20	65	67	75
		20p	0	64	7	7	13	12	100	100	100	18	0	60	16	16	18	18	100	81	100	100	0	89	68	68	28	50	28	73	75	81
		40p	0	69	49	49	27	19	100	100	100	42	0	71	58	58	39	39	100	86	100	100	0	94	99	99	48	69	48	79	79	84
5	25	3p	0	2	0	0	1	1	100	77	1	0	0	5	0	0	1	1	72	37	7	7	0	57	0	0	1	0	1	52	33	55
		5p	0	22	0	0	2	1	100	86	41	0	0	34	0	0	6	6	73	47	21	21	0	88	0	0	14	3	14	67	46	63
		10p	0	37	0	0	7	4	100	91	92	6	0	75	0	0	38	38	75	58	37	37	0	91	0	0	63	37	63	79	61	72
		20p	0	28	0	0	25	14	100	95	99	37	0	85	0	0	79	79	77	68	51	51	0	97	0	0	94	69	94	87	72	79
		40p	0	33	0	0	55	38	100	98	100	69	0	93	0	0	90	90	81	75	64	64	0	100	0	0	100	91	100	92	80	85
5	50	3p	0	0	0	0	4	0	90	41	0	0	0	0	0	0	21	21	27	78	25	25	0	0	0	0	60	40	60	100	93	100
		5p	0	0	0	0	23	4	91	49	2	0	0	0	0	0	66	66	44	89	50	50	0	5	0	0	96	90	96	100	95	100
		10p	0	2	0	0	77	42	92	60	27	0	0	1	0	0	99	99	67	96	78	78	0	99	0	0	100	100	100	100	97	100
		20p	0	81	0	0	93	91	92	68	60	46	0	68	0	0	100	100	79	98	90	90	0	100	0	0	100	100	100	100	98	100
		40p	0	95	0	0	98	100	92	75	82	98	0	100	0	0	100	100	85	99	95	95	0	100	0	0	100	100	100	100	99	100
7	21	3p	0	5	10	1	1	1	100	90	3	0	0	22	80	0	0	0	100	58	5	5	0	43	90	0	0	0	0	32	15	44
		5p	0	25	10	0	1	1	100	97	71	0	0	38	80	1	0	0	100	64	42	42	0	45	90	0	0	0	0	36	30	54
		10p	0	31	10	1	2	1	100	100	100	1	0	36	80	1	3	3	100	73	88	88	0	52	90	0	5	3	5	44	53	66
		20p	0	22	10	1	8	3	100	100	100	13	0	38	80	1	19	19	100	81	99	99	0	62	90	6	23	16	23	53	73	76
		40p	0	24	10	1	29	12	100	100	100	42	0	49	80	4	49	49	100	88	100	100	0	72	90	36	45	31	45	61	85	83
7	35	3p	0	0	100	0	0	0	100	76	0	0	0	0	100	0	0	0	90	27	0	0	0	17	100	0	0	0	0	44	41	64
		5p	0	9	100	0	0	0	100	89	13	0	0	3	100	0	4	4	90	35	8	8	0	79	100	0	3	0	3	53	55	72
		10p	0	31	100	0	4	0	100	97	88	1	0	49	100	0	32	32	91	46	39	39	0	82	100	0	38	11	38	62	72	80
		20p	0	13	100	0	30	0	100	99	94	24	0	83	100	0	60	60	92	57	66	66	0	82	100	0	78	51	78	71	81	87
		40p	0	13	100	0	64	8	100	100	97	56	0	86	100	0	82	82	93	66	83	83	0	83	100	0	91	77	91	79	86	91
7	70	3p	0	0	100	0	0	0	59	33	0	0	0	0	100	0	10	10	35	74	25	25	0	0	100	0	39	24	39	100	85	99
		5p	0	0	100	0	6	0	61	38	0	0	0	0	100	0	41	41	51	84	50	50	0	0	100	0	85	74	85	100	91	99
		10p	0	0	100	0	70	26	63	47	11	0	0	0	100	0	87	87	64	91	78	78	0	54	100	0	100	99	100	100	95	99
		20p	0	17	100	0	98	78	67	56	36	43	0	2	100	0	96	96	75	95	87	87	0	100	100	0	100	100	100	100	98	99
		40p	0	100	100	0	100	96	79	62	51	100	0	93	100	0	100	100	80	97	90	90	0	100	100	0	100	100	100	100	100	99

Table 15
Proportion of Samples Agreeing with Rule Applied to Population R-Matrix, Phi = Mixed

			Low Communality										Wide Communality										High Communality									
k	p	Size	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR	CNG	SE Scree	MR	T	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
3	9	3p	0	38	99	99	25	30	68	93	53	9	0	53	99	99	20	20	84	69	53	53	0	70	99	99	18	26	18	72	67	85
		5p	0	54	100	100	29	32	80	98	91	16	0	68	100	100	26	26	88	74	74	74	0	77	100	100	27	33	27	81	73	90
		10p	0	62	100	100	36	40	93	100	100	37	0	78	100	100	41	41	92	78	91	91	0	83	100	100	47	46	47	88	79	95
		20p	0	66	100	100	47	54	99	100	100	67	0	85	100	100	61	61	95	82	97	97	0	87	100	100	70	60	70	92	84	98
		40p	0	68	100	100	58	70	100	100	100	84	0	89	100	100	79	79	97	85	99	99	0	90	100	100	85	76	85	94	87	100
3	15	3p	0	21	0	0	43	36	70	53	14	0	0	38	0	0	25	25	49	70	66	66	0	91	0	0	53	38	53	84	67	79
		5p	0	52	0	0	54	46	75	60	34	3	0	64	0	0	41	41	57	76	83	83	0	99	0	0	85	70	85	92	77	82
		10p	0	73	1	1	72	67	79	66	56	24	0	72	2	2	71	71	68	81	91	91	0	100	13	13	100	97	100	97	87	87
		20p	0	80	28	28	87	79	82	71	73	78	0	75	36	36	93	93	77	83	96	96	0	100	81	81	100	100	100	98	94	92
		40p	0	85	90	90	97	87	87	77	84	92	0	84	91	91	99	99	88	86	99	99	0	100	100	100	100	100	100	99	98	96
3	30	3p	0	0	0	0	65	55	88	64	6	0	0	0	0	0	88	88	57	93	89	89	0	10	0	0	100	99	100	100	100	100
		5p	0	1	0	0	84	76	93	71	34	0	0	1	0	0	96	96	68	96	97	97	0	84	0	0	100	100	100	100	100	100
		10p	0	57	0	0	96	96	97	78	67	0	0	55	0	0	100	100	77	97	99	99	0	100	0	0	100	100	100	100	100	100
		20p	0	100	0	0	98	100	99	83	83	56	0	99	0	0	100	100	83	99	99	99	0	100	0	0	100	100	100	100	100	100
		40p	0	100	0	0	99	100	100	88	89	100	0	100	0	0	100	100	90	100	99	99	0	100	0	0	100	100	100	100	100	100
5	15	3p	0	19	0	0	12	17	97	82	7	2	0	29	0	0	3	3	61	57	33	33	0	58	0	0	3	3	3	50	58	74
		5p	0	39	0	0	15	19	99	89	56	4	0	45	0	0	6	6	66	65	58	58	0	66	0	0	11	10	11	59	67	81
		10p	0	42	0	0	23	29	100	93	89	19	0	59	1	1	13	13	71	72	76	76	0	73	10	10	29	35	29	70	76	89
		20p	0	41	13	13	34	45	100	95	95	57	0	67	16	16	29	29	76	79	86	86	0	80	76	76	43	63	43	79	83	94
		40p	0	44	65	65	48	69	100	95	98	83	0	75	67	67	50	50	81	85	90	90	0	86	100	100	59	86	59	84	88	97
5	25	3p	0	0	0	0	11	10	54	57	2	0	0	1	0	0	32	32	42	41	21	21	0	51	0	0	4	1	4	61	93	79
		5p	0	13	0	0	16	19	55	62	28	0	0	23	0	0	62	62	56	55	36	36	0	91	0	0	18	6	18	74	94	87
		10p	0	67	0	0	27	48	56	72	60	2	0	82	0	0	87	87	73	69	53	53	0	94	0	0	70	31	70	84	95	94
		20p	0	87	0	0	40	80	58	80	80	48	0	93	0	0	96	96	87	79	68	68	0	94	0	0	93	79	93	91	97	95
		40p	0	96	0	0	62	96	64	87	90	90	0	94	0	0	99	99	95	83	81	81	0	96	0	.0	100	91	100	96	99	96
5	50	3p	0	0	0	0	8	1	45	47	0	0	0	0	0	0	74	74	57	96	81	81	0	0	0	0	87	78	87	100	99	100
		5p	0	0	0	0	34	8	51	53	12	0	0	0	0	0	97	97	71	100	88	88	0	3	0	0	100	98	100	100	98	100
		10p	0	2	0	0	85	50	60	61	63	0	0	0	0	0	100	100	81	100	94	94	0	99	0	0	100	100	100	100	98	100
		20p	0	83	0	0	99	88	70	71	82	42	0	64	0	0	100	100	88	100	97	97	0	100	0	0	100	100	100	100	97	100
		40p	0	100	0	0	100	100	82	80	91	100	0	100	0	0	100	100	93	100	98	98	0	100	0	0	100	100	100	100	97	100
7	21	3p	0	7	10	2	8	7	96	61	1	0	0	17	10	0	1	1	32	49	21	21	0	49	90	1	0	0	0	39	38	64
		5p	0	30	10	2	10	9	97	66	18	1	0	38	10	0	4	4	40	61	47	47	0	49	90	1	2	0	2	50	46	71
		10p	0	38	10	2	20	13	99	73	59	8	0	52	10	0	17	17	47	72	70	70	0	54	90	0	22	3	22	64	53	79
		20p	0	35	10	1	38	20	100	79	79	31	0	60	10	0	40	40	52	82	83	83	0	65	90	4	64	21	64	77	59	84
		40p	0	41	10	2	57	33	100	83	85	59	0	68	10	0	67	67	62	89	89	89	0	76	90	36	81	52	81	87	64	87
7	35	3p	0	0	100	1	7	1	38	39	0	0	0	0	100	0	1	1	39	39	14	14	0	13	100	1	0	0	0	68	36	62
		5p	0	2	100	0	16	2	38	45	2	0	0	5	100	0	5	5	51	55	48	48	0	73	100	0	3	0	3	78	53	68
		10p	0	47	100	0	37	9	36	52	27	1	0	52	100	0	36	36	63	71	76	76	0	75	100	0	12	13	12	86	66	76
		20p	0	79	100	0	57	40	37	59	59	64	0	80	100	0	69	69	73	83	86	86	0	78	100	0	42	22	42	91	75	81
		40p	0	87	100	0	65	77	40	68	79	90	0	80	100	0	88	88	82	89	92	92	0	86	100	0	87	40	87	93	81	87
7	70	3p	0	0	100	0	1	0	76	45	0	0	0	0	100	0	0	0	75	59	56	56	0	0	100	0	6	2	6	98	62	84
		5p	0	0	100	0	15	2	82	52	2	0	0	0	100	0	6	6	75	74	91	91	0	0	100	0	34	16	34	99	72	87
		10p	0	0	100	0	71	33	88	63	47	0	0	0	100	0	73	73	74	88	93	93	0	51	100	0	92	82	92	100	81	87
		20p	0	17	100	0	99	77	93	71	83	48	0	2	100	0	90	90	75	95	97	97	0	100	100	0	100	92	100	.100	88	88
		40p	0	100	100	0	100	97	97	78	95	100	0	96	100	0	92	92	80	98	99	99	0	100	100	0	100	100	100	100	93	89

Table 16
Omega Squared for Proportion of Samples Agreeing with Rule Applied to Population R-Matrix.

					Parallel Analysis					
Effect	CNG	SE_{scree}	MR	t	PA_{MH}	PA_{CL}	PA_{LL}	MAP	KSR	M-KSR
k	0.000	0.076	0.458	0.238	0.040	0.055	0.003	0.058	0.040	0.013
p/k	0.000	0.040	0.064	0.281	0.276	0.239	0.015	0.095	0.036	0.005
N	0.000	0.359	0.023	0.051	0.197	0.168	0.071	0.102	0.256	0.255
Communality	0.000	0.135	0.009	0.005	0.033	0.036	0.017	0.006	0.078	0.361
Phi	0.000	0.006	0.002	0.003	0.086	0.099	0.010	0.002	0.016	0.011
k X p/k	0.000	0.001	0.246	0.213	0.003	0.001	0.002	0.010	0.004	0.004
k X N	0.000	0.009	0.011	0.013	0.007	0.004	0.000	0.003	0.011	0.000
k X Comm	0.000	0.001	0.002	0.001	0.001	0.001	0.004	0.000	0.004	0.001
k X Phi	0.000	0.001	0.003	0.001	0.009	0.013	0.005	0.001	0.003	0.001
p/k X N	0.000	0.086	0.012	0.026	0.017	0.009	0.001	0.010	0.013	0.005
p/k X Comm	0.000	0.009	0.009	0.004	0.004	0.004	0.055	0.170	0.049	0.008
p/k X Phi	0.000	0.004	0.004	0.002	0.008	0.015	0.053	0.030	0.024	0.007
N X Comm	0.000	0.013	0.002	0.006	0.001	0.001	0.005	0.001	0.063	0.123
N X Phi	0.000	0.001	0.000	0.001	0.013	0.009	0.000	0.001	0.007	0.000
Comm X Phi	0.000	0.001	0.001	0.000	0.002	0.004	0.035	0.015	0.006	0.000
k X p/k X N	0.000	0.026	0.057	0.103	0.004	0.005	0.000	0.000	0.001	0.000
k X p/k X Comm	0.000	0.003	0.012	0.008	0.001	0.002	0.009	0.008	0.002	0.001
k X N X Comm	0.000	0.012	0.001	0.002	0.000	0.000	0.000	0.000	0.002	0.003
p/k X N X Comm	0.000	0.031	0.001	0.005	0.010	0.009	0.003	0.014	0.006	0.030
k X p/k X Phi	0.000	0.001	0.007	0.002	0.006	0.004	0.010	0.005	0.001	0.001
k X N X Phi	0.000	0.001	0.000	0.000	0.002	0.001	0.000	0.000	0.000	0.000
p/k X N X Phi	0.000	0.003	0.000	0.001	0.019	0.017	0.009	0.002	0.006	0.000
Comm X p/k X Phi	0.000	0.008	0.003	0.000	0.005	0.011	0.068	0.077	0.046	0.002
Comm X k X Phi	0.000	0.001	0.003	0.000	0.002	0.001	0.010	0.004	0.005	0.001
Comm X n X Phi	0.000	0.001	0.000	0.000	0.001	0.002	0.000	0.000	0.003	0.002
k X p/k X N X Comm	0.000	0.032	0.005	0.011	0.001	0.001	0.000	0.000	0.000	0.000
k X p/k X N X Phi	0.000	0.001	0.000	0.001	0.007	0.005	0.000	0.000	0.001	0.000
Comm X k X p/k X Phi	0.000	0.001	0.005	0.001	0.005	0.005	0.017	0.011	0.006	0.002
Comm X p/k X n X Phi	0.000	0.004	0.000	0.000	0.009	0.012	0.005	0.003	0.010	0.002
Comm X k X n X Phi	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
k X p/k X N X Comm X phi	0.000	0.000	0.000	0.001	0.002	0.003	0.000	0.000	0.001	0.000

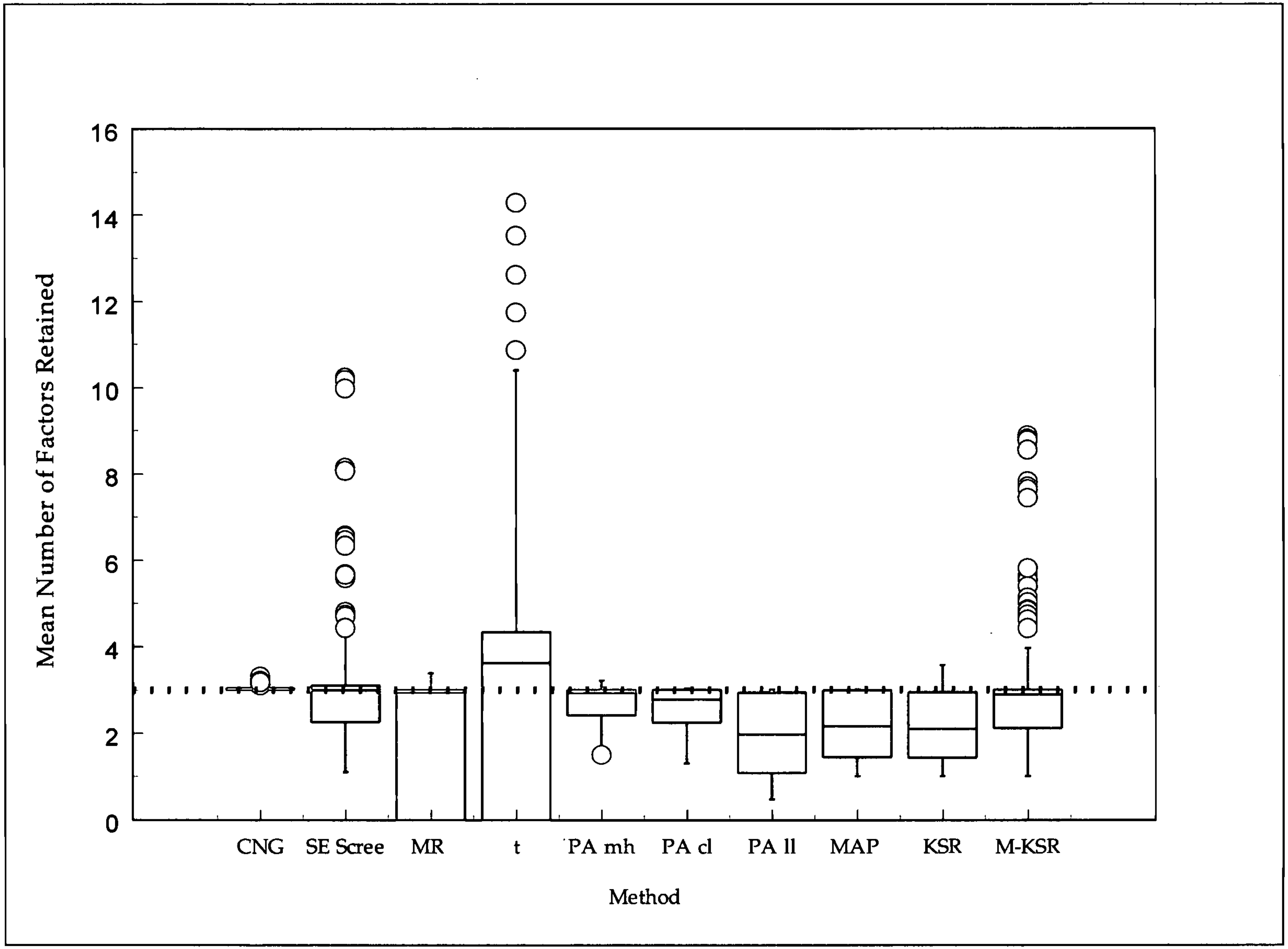

Figure 1. Distributions of Mean Number of Factors Retained, k = 3

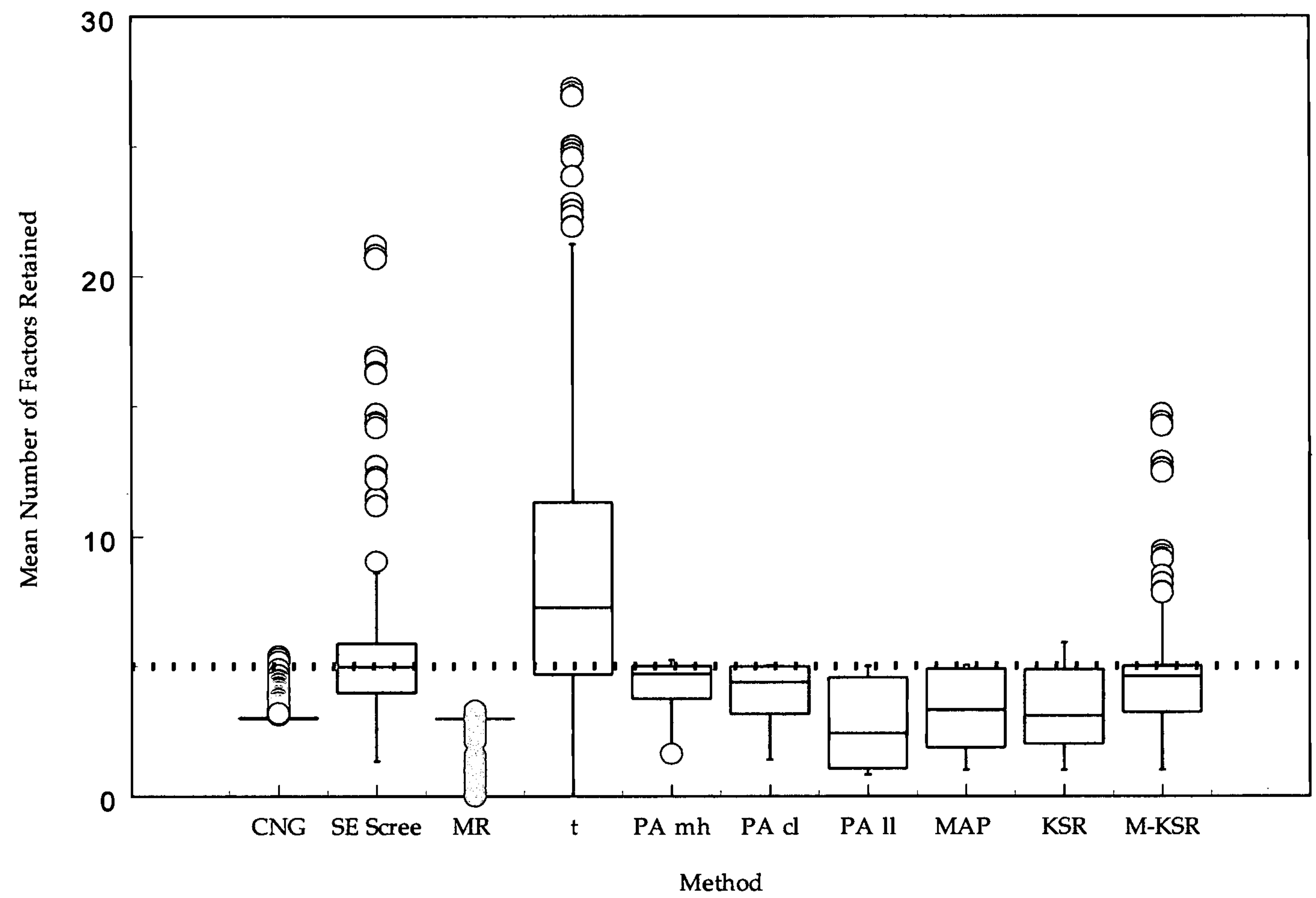

Figure 2. Distributions of Mean Number of Factors Retained, k = 5

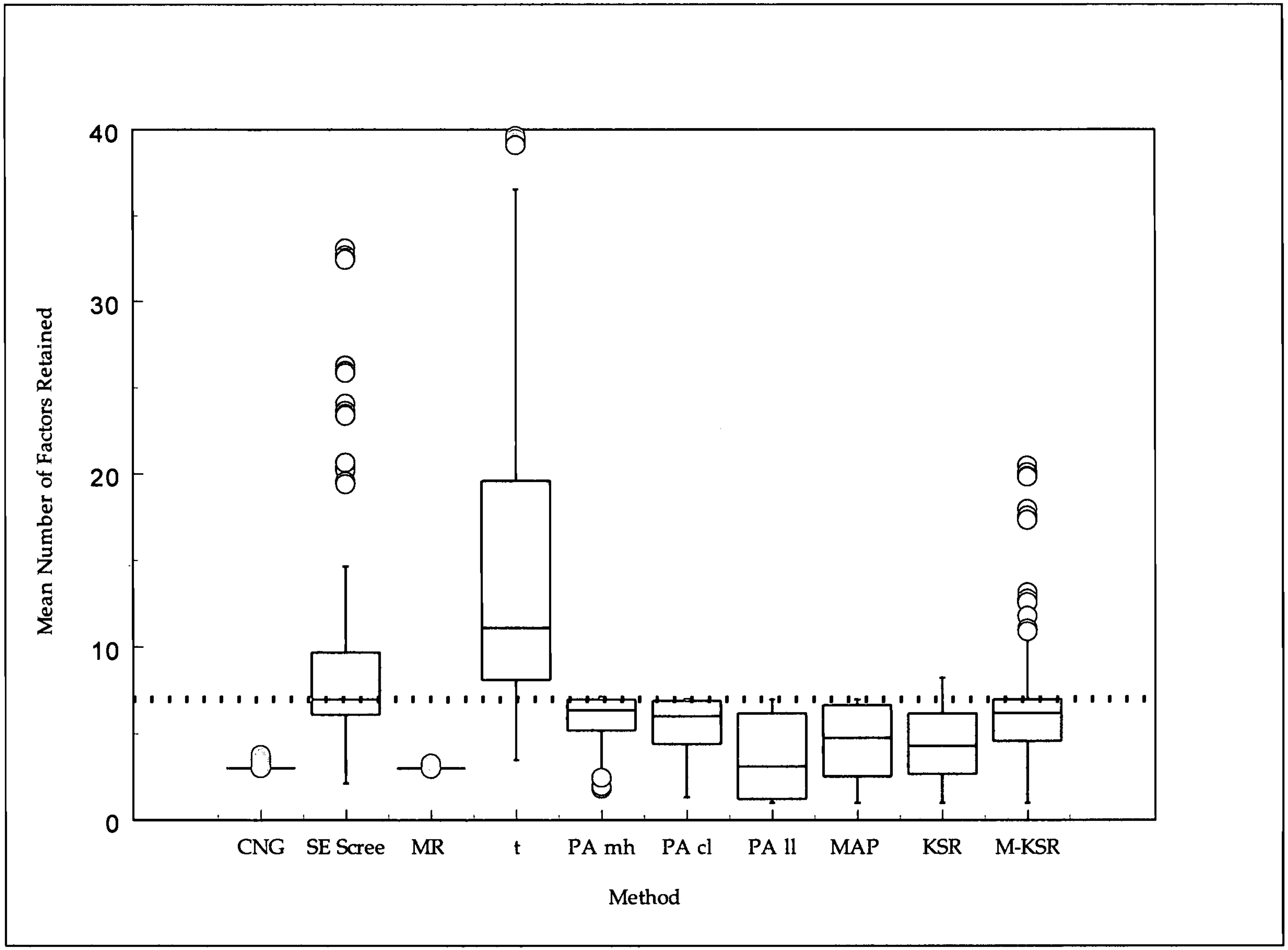

Figure 3. Distributions of Mean Number of Factors Retained, k = 7

Figure 4. Mean Number of Factors Retained by True Number of Factors

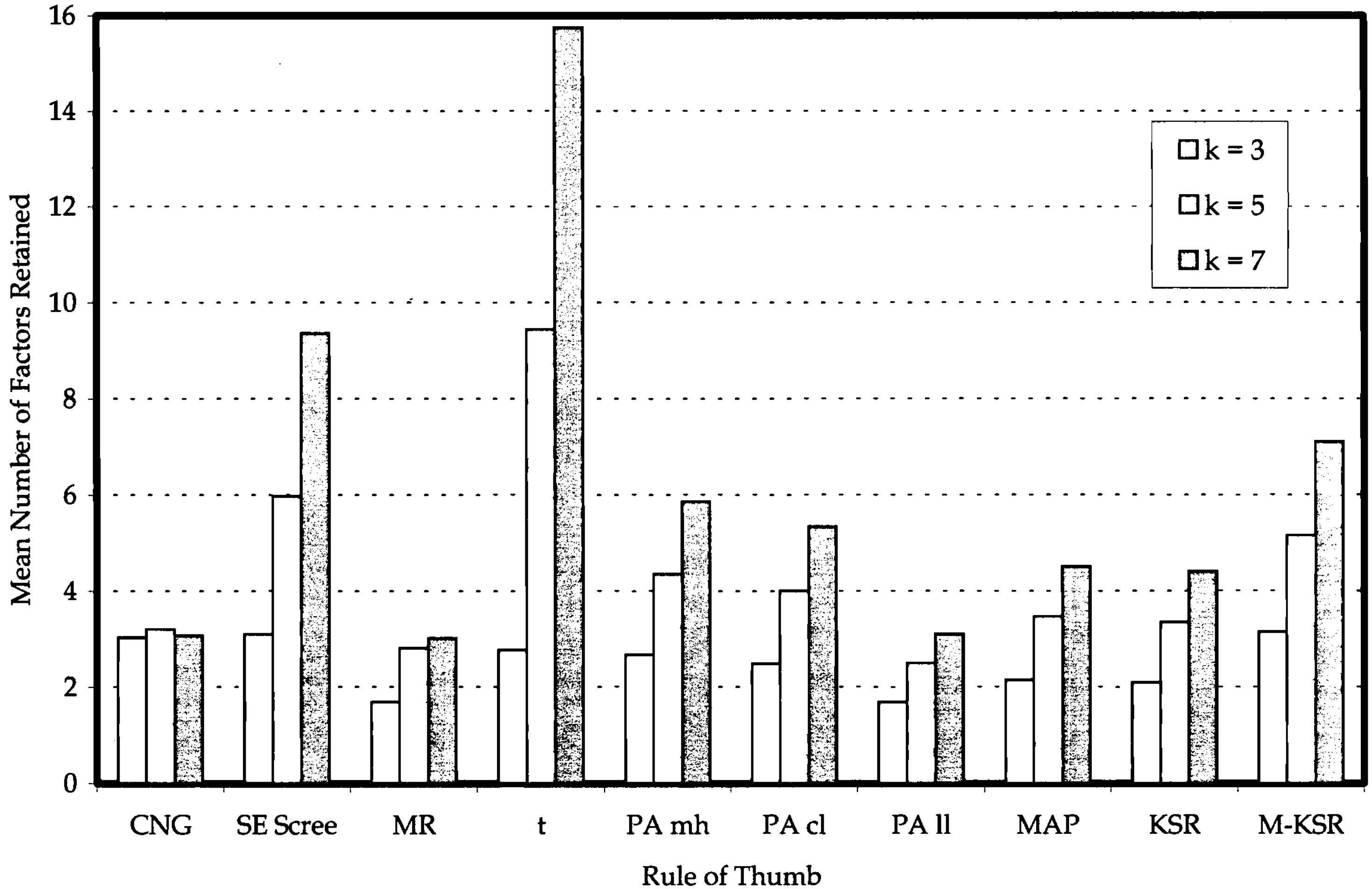

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Figure 5. Mean Number of Factors Retained by N per Variable (N:p) for k = 5

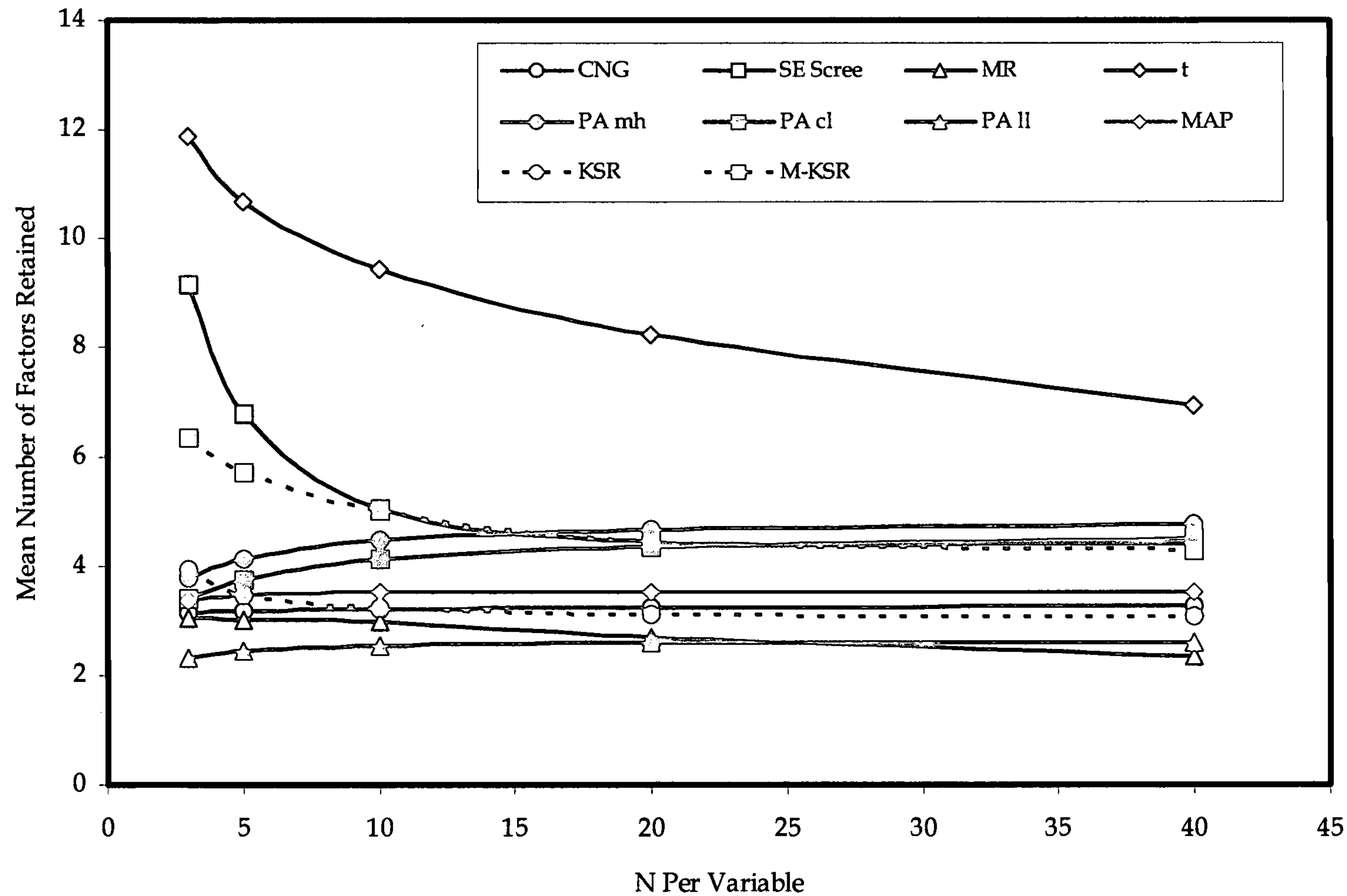

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Figure 6. Mean Number of Factors Retained by Number of Variables Per Factor (p:k) for k = 5

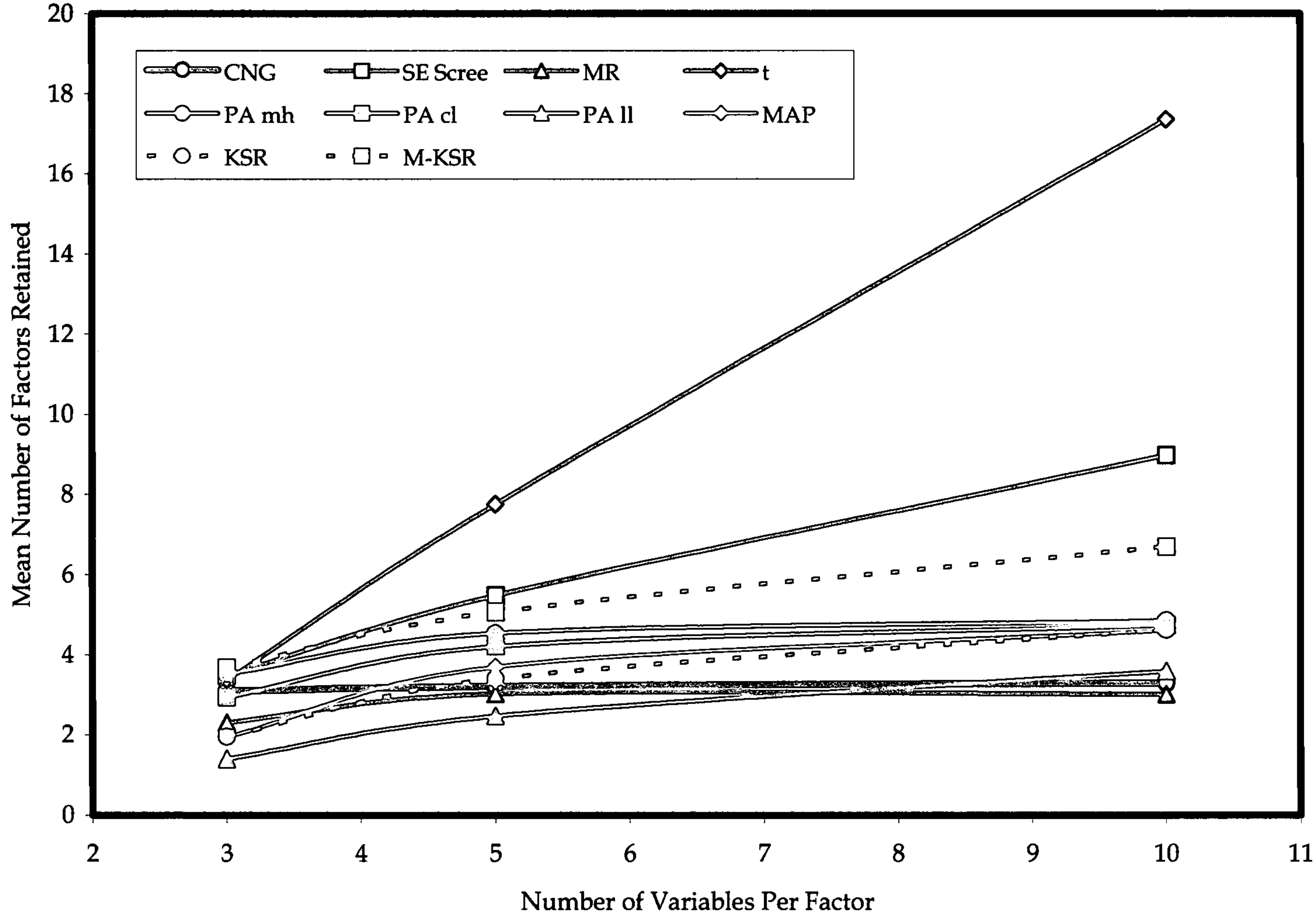

Figure 7. Mean Number of Factors Retained by Communality Type for k = 5

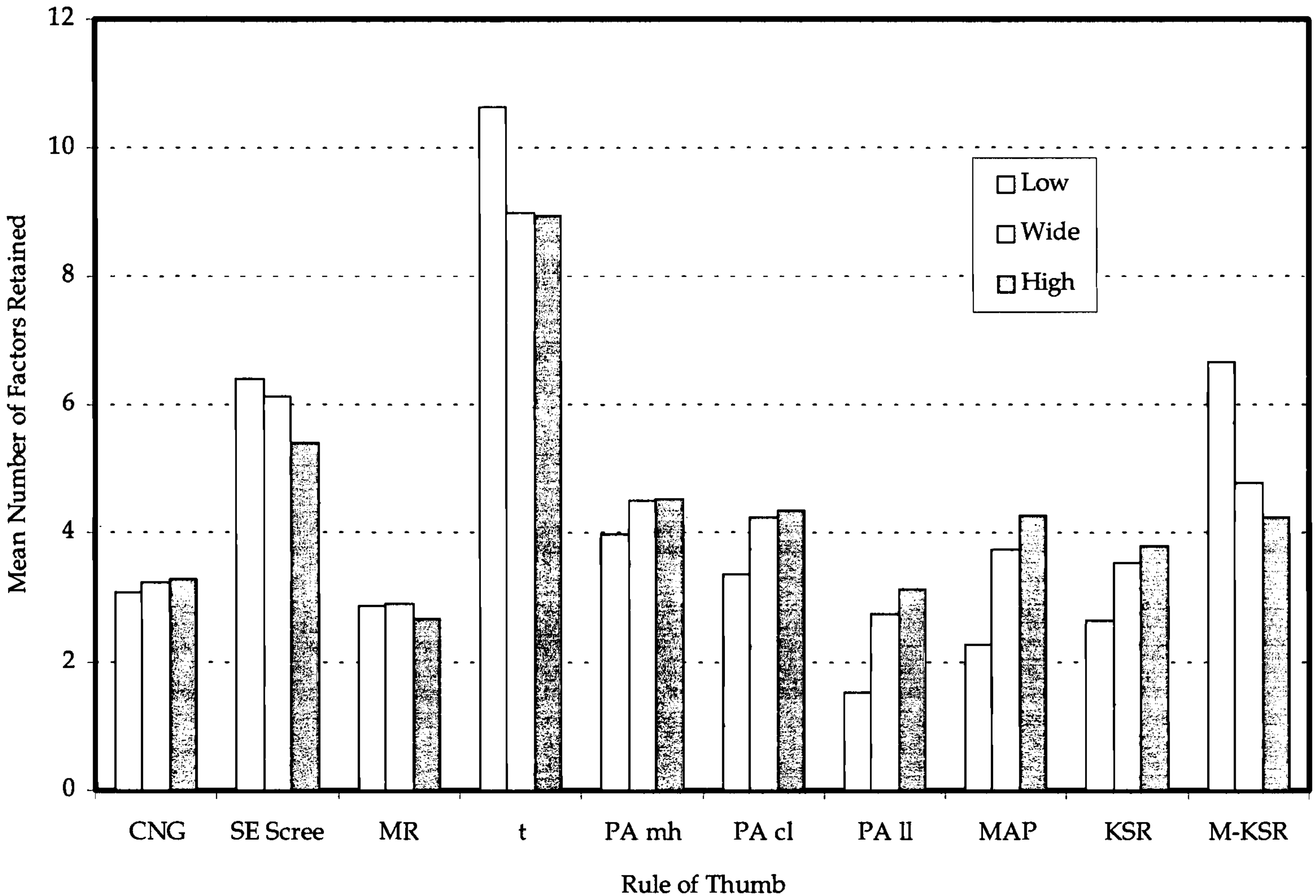

Figure 8. Mean Number of Factors Retained by Phi for k = 5

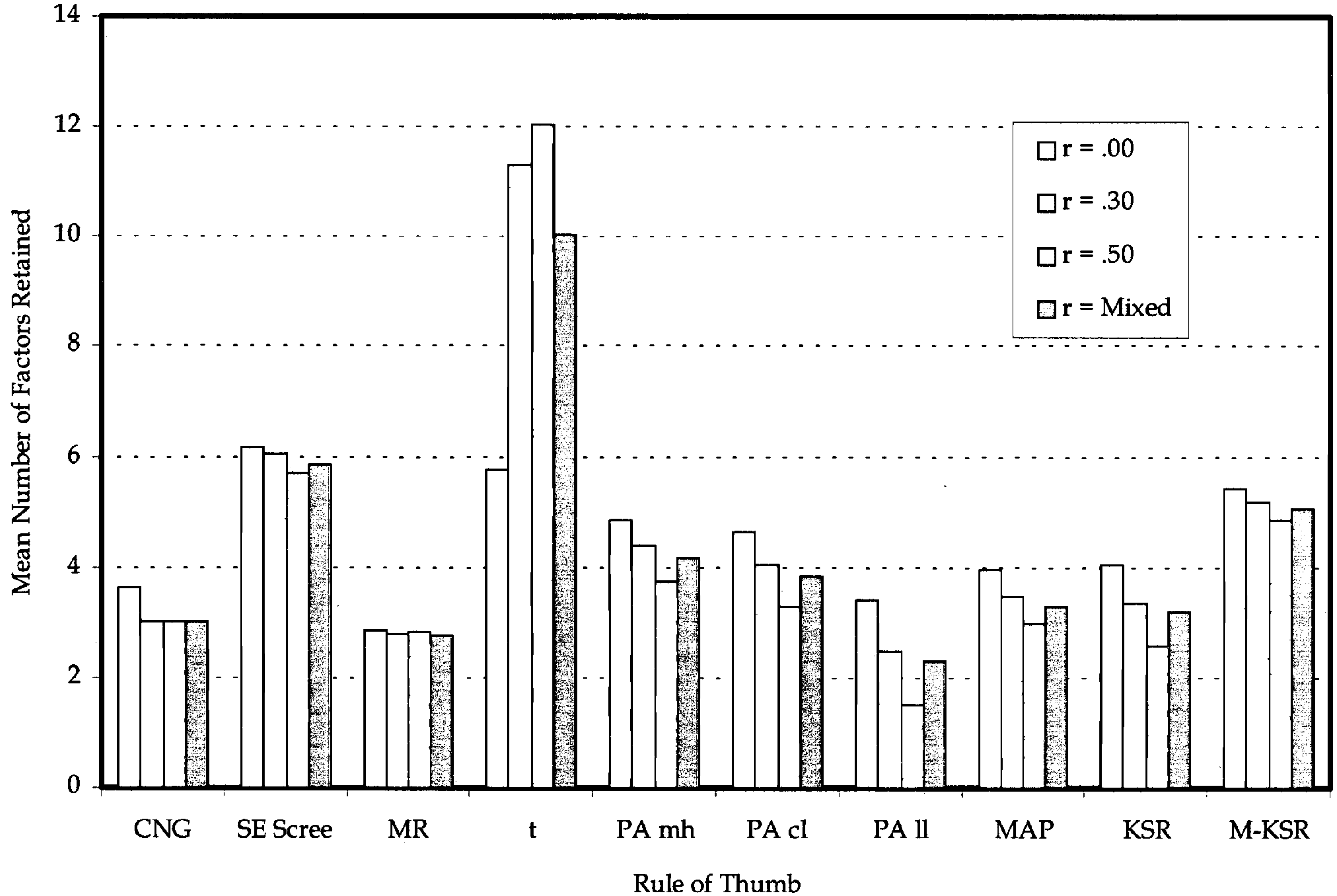

Figure 9. Proportion of Samples Retaining K Factors by True Number of Factors

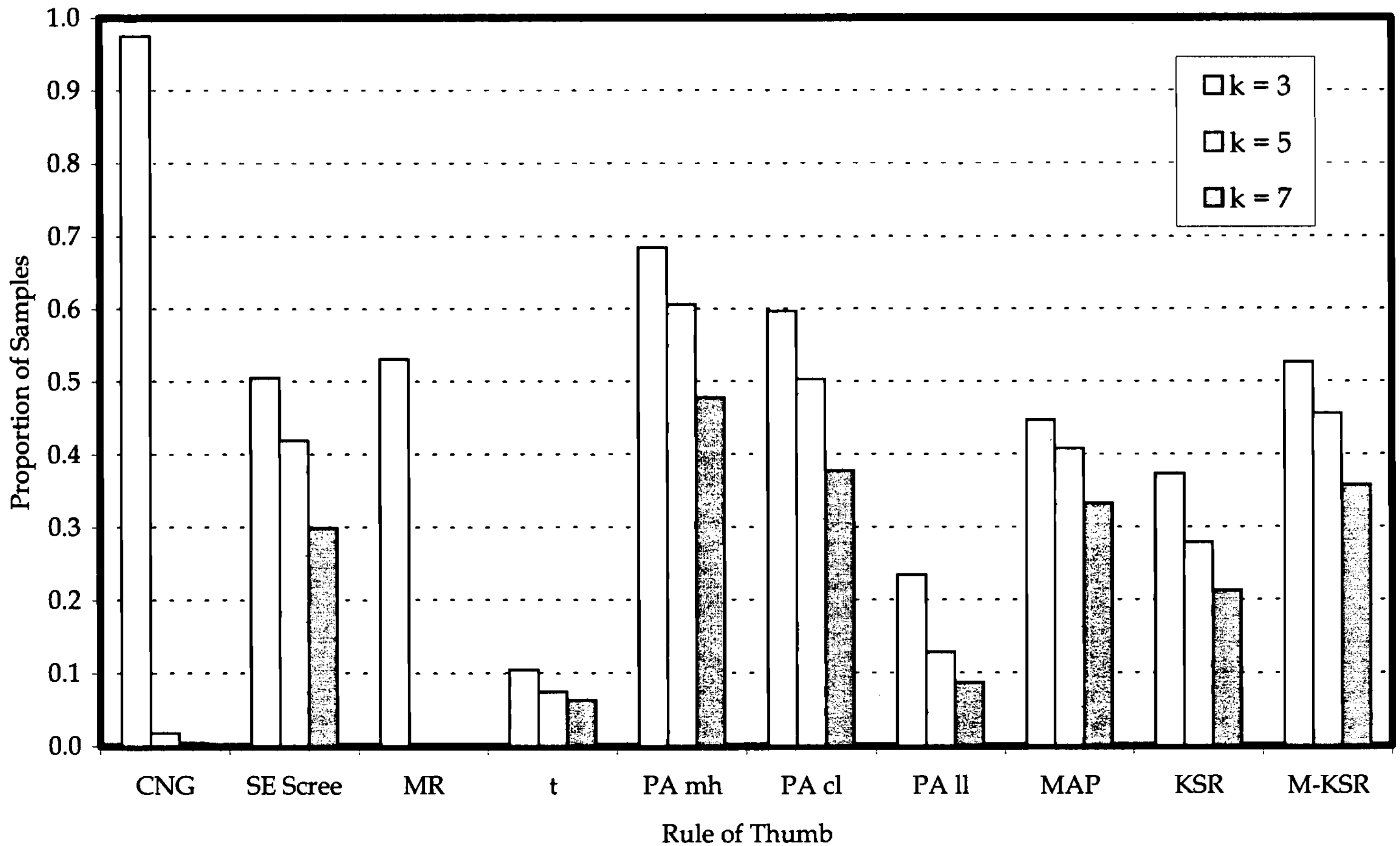

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Figure 10. Proportion of Samples Retaining K Factors by N per Variable (N:p)

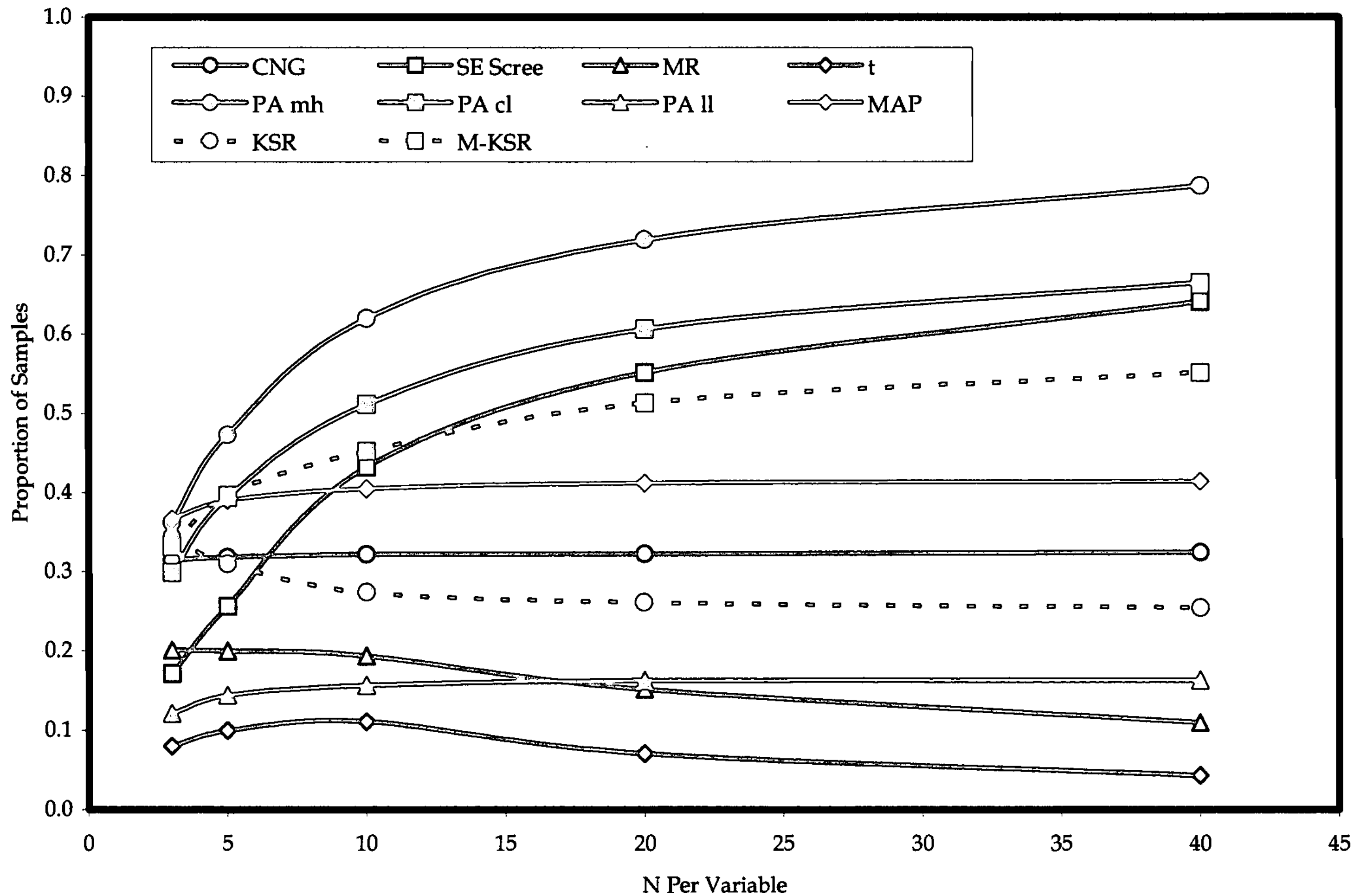

Figure 11. Proportion of Samples Retaining K Factors by Number of Variables Per Factor (p:k)

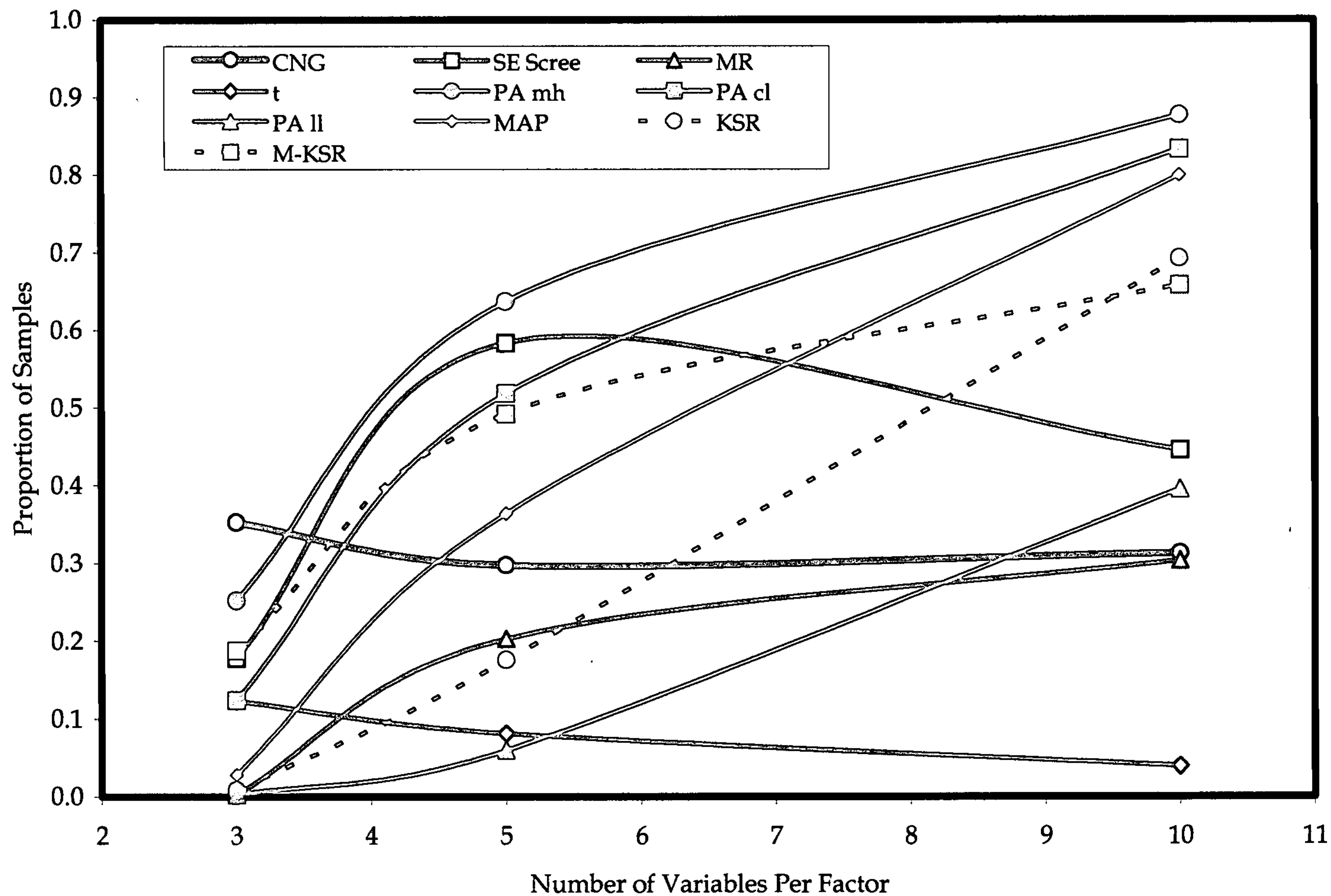

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Figure 12. Proportion of Samples Retaining K Factors by Communality Type

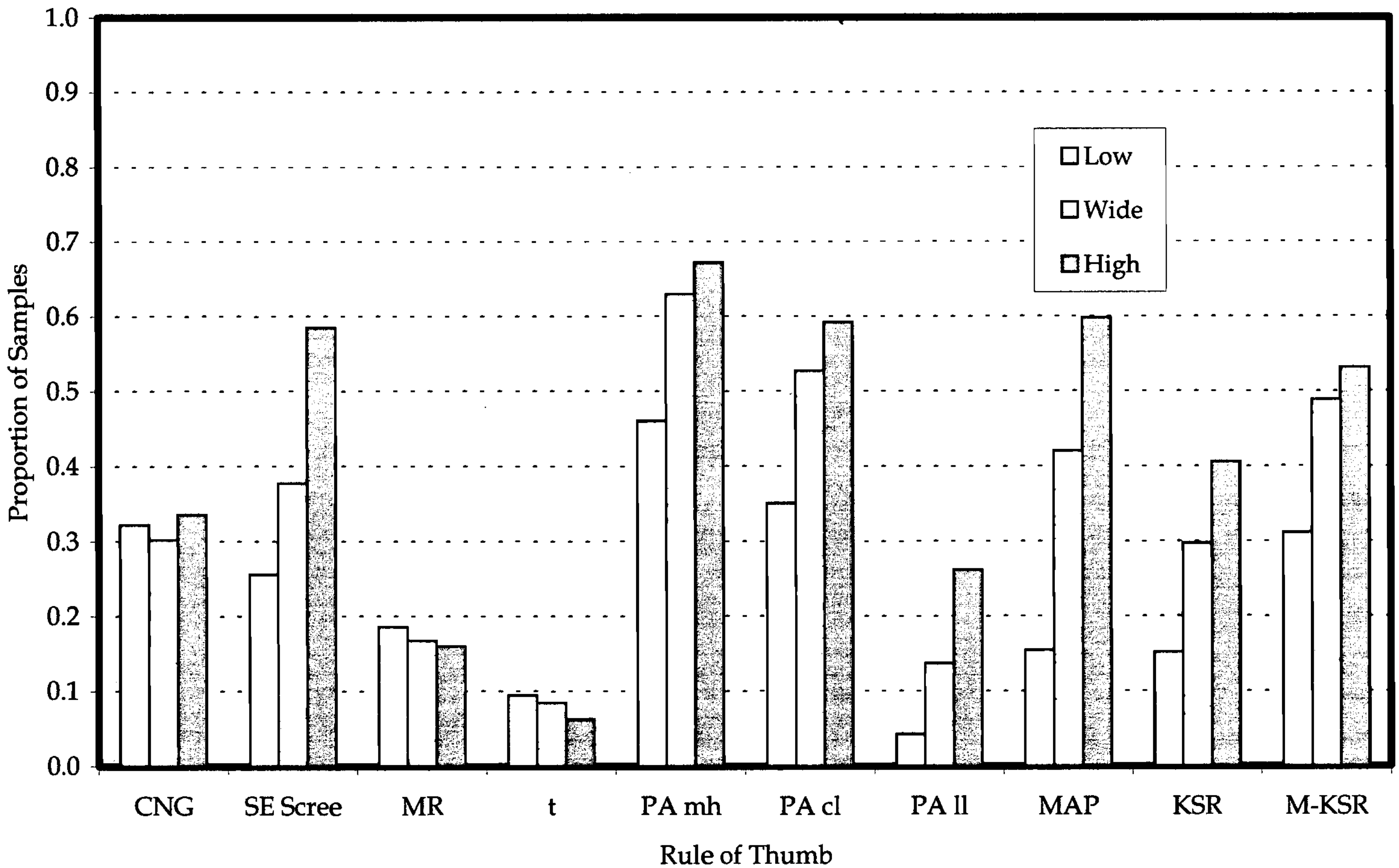

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Figure 13. Proportion of Samples Retaining K Factors by Phi

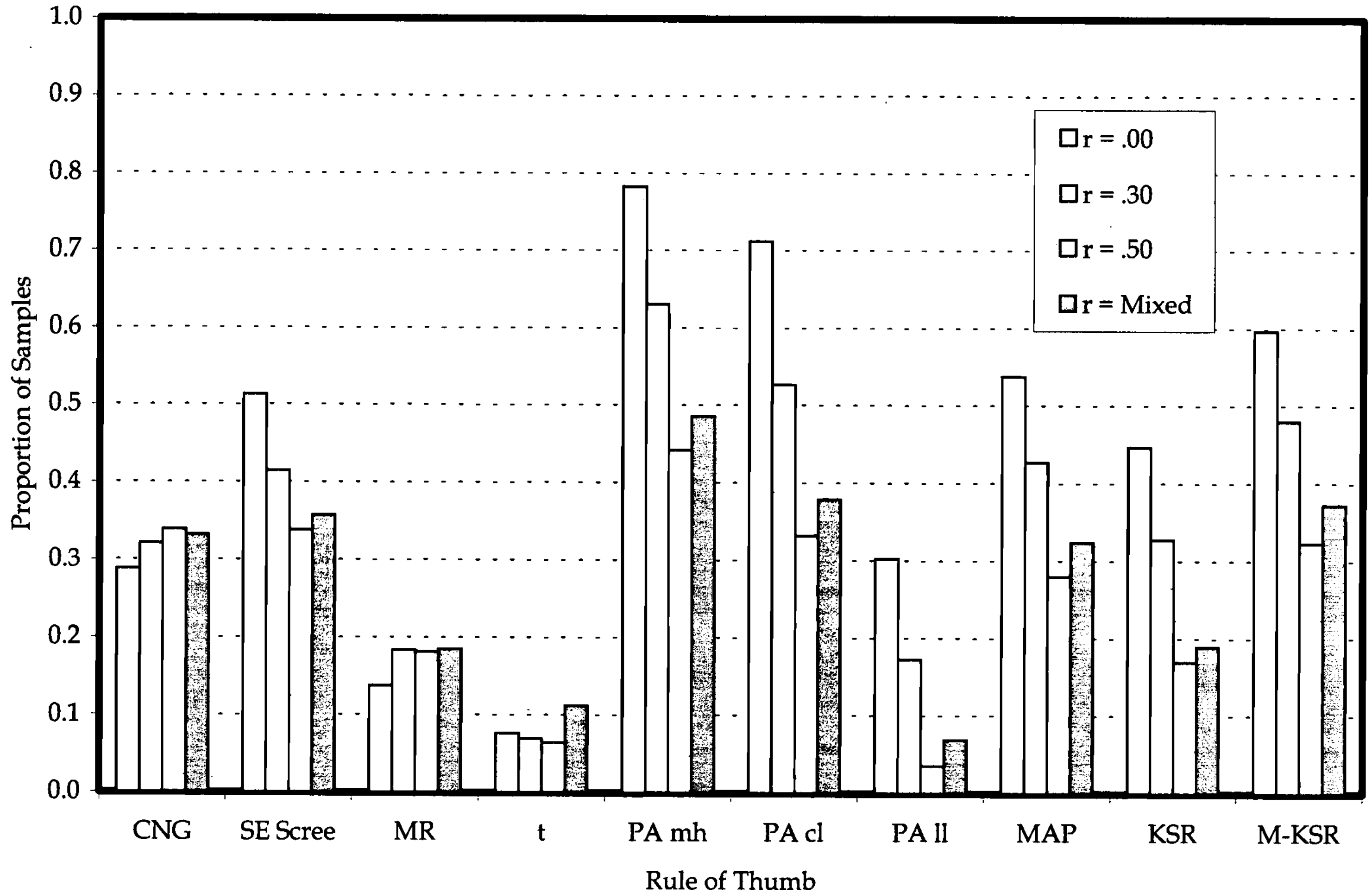


Figure 14. Proportion of Samples Agreeing with Rule Applied to Population R-Matrix by True Number of Factors

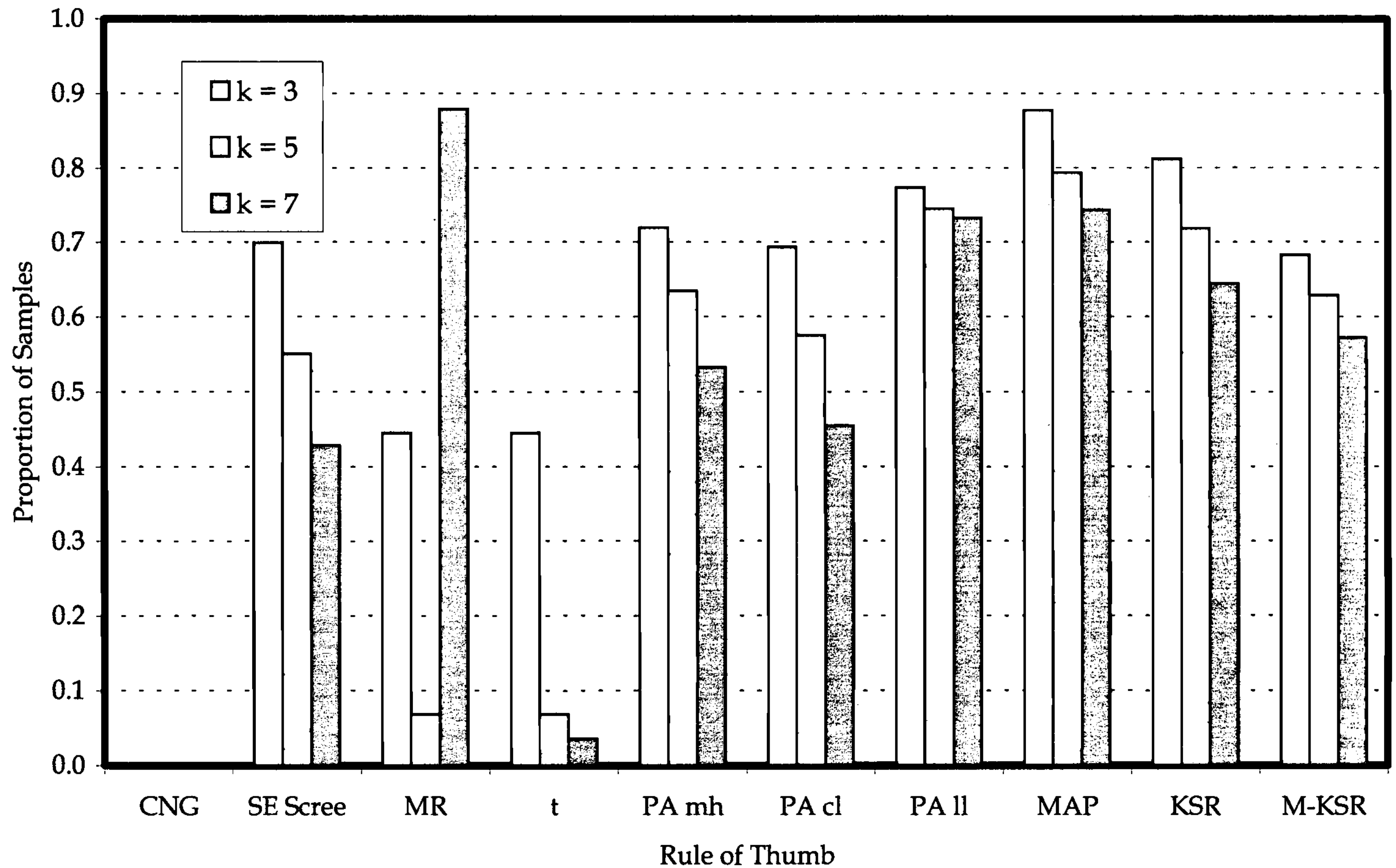


Figure 15. Proportion of Samples Agreeing with Rule Applied to Population R-Matrix by N per Variable (N:p)

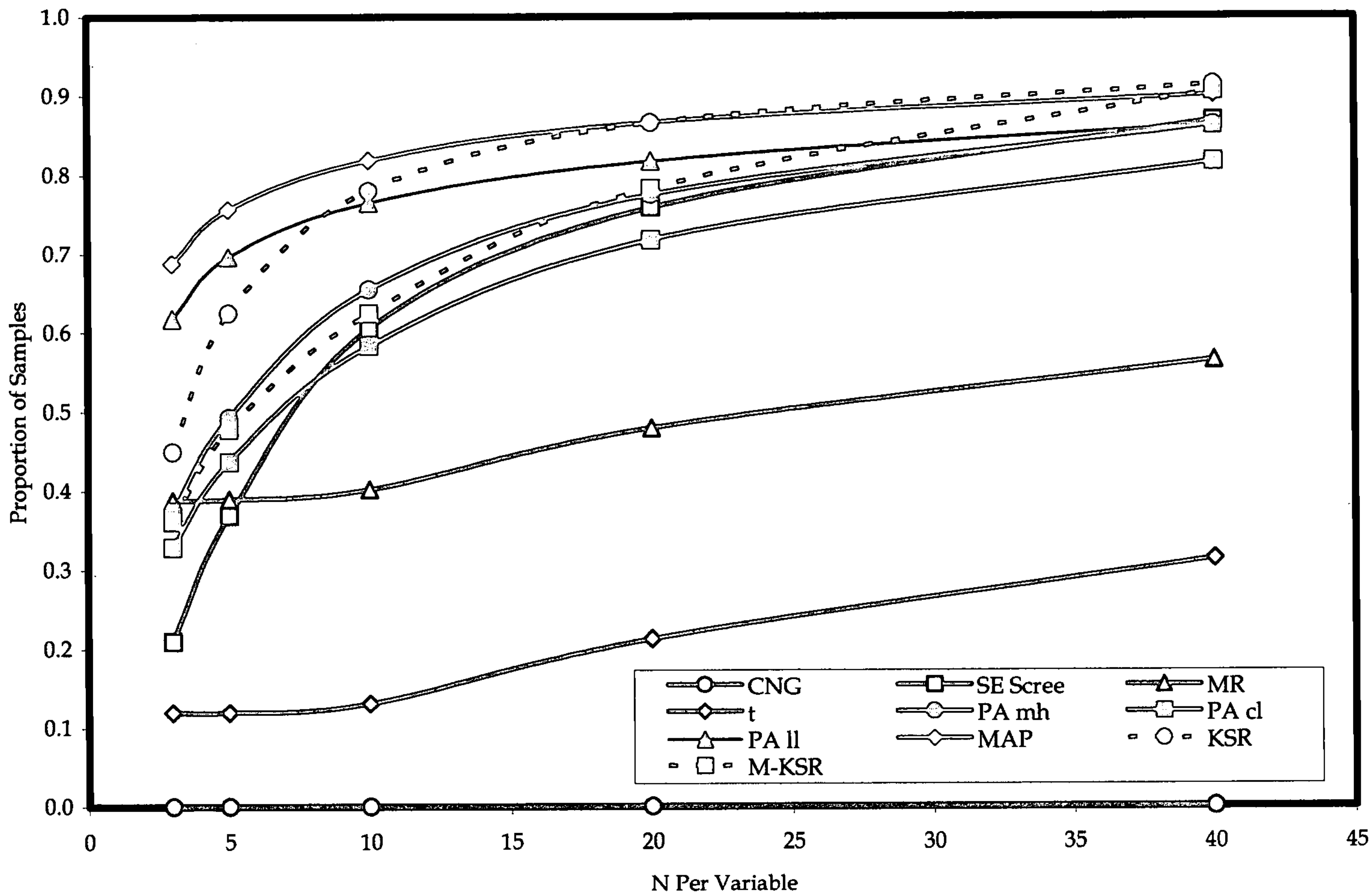


Figure 16. Proportion of Samples Agreeing with Rule Applied to Population R-Matrix by Number of Variables Per Factor (p:k)

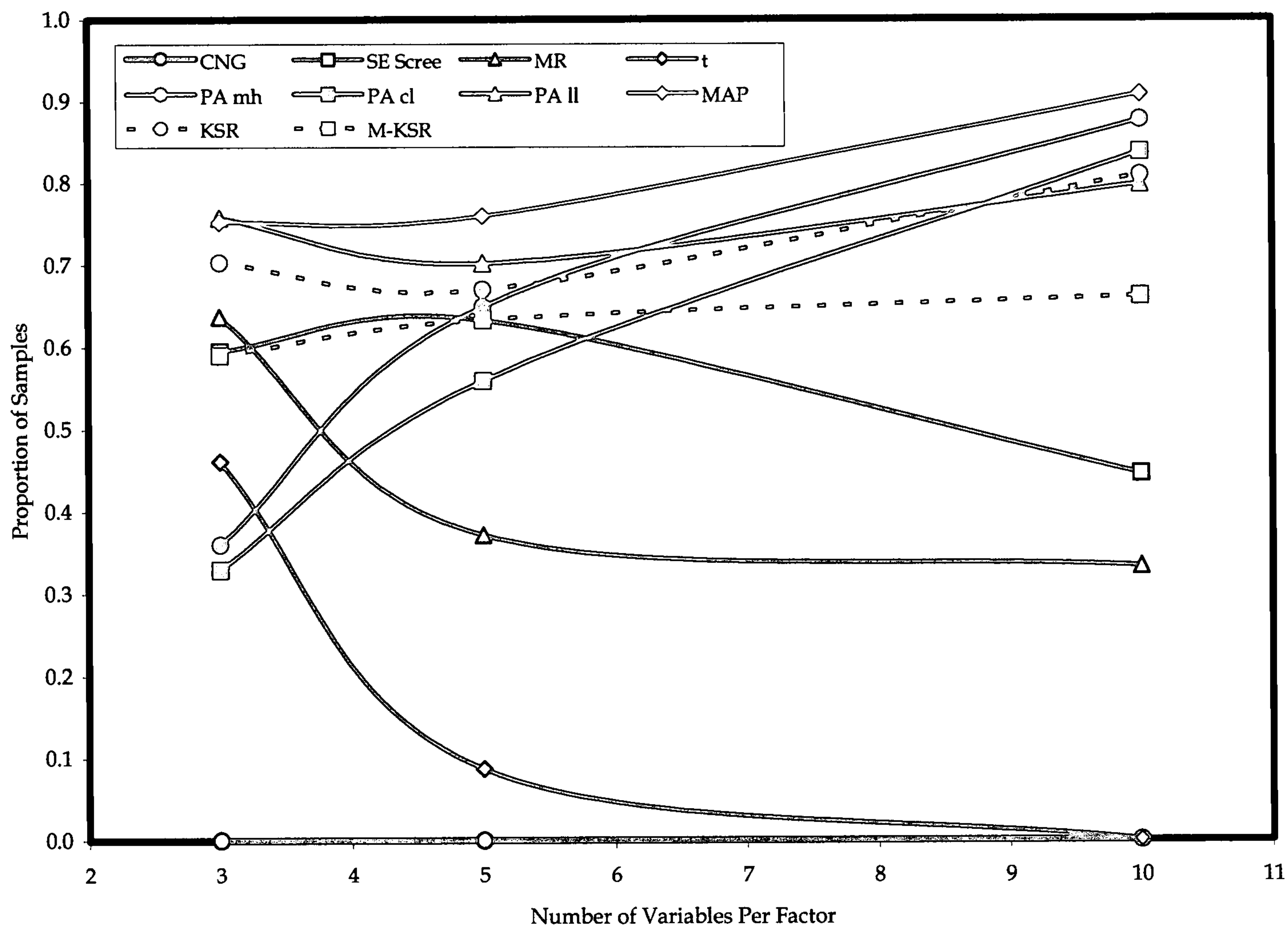


Figure 17. Proportion of Samples Agreeing with Rule Applied to Population R-Matrix by Communality Type

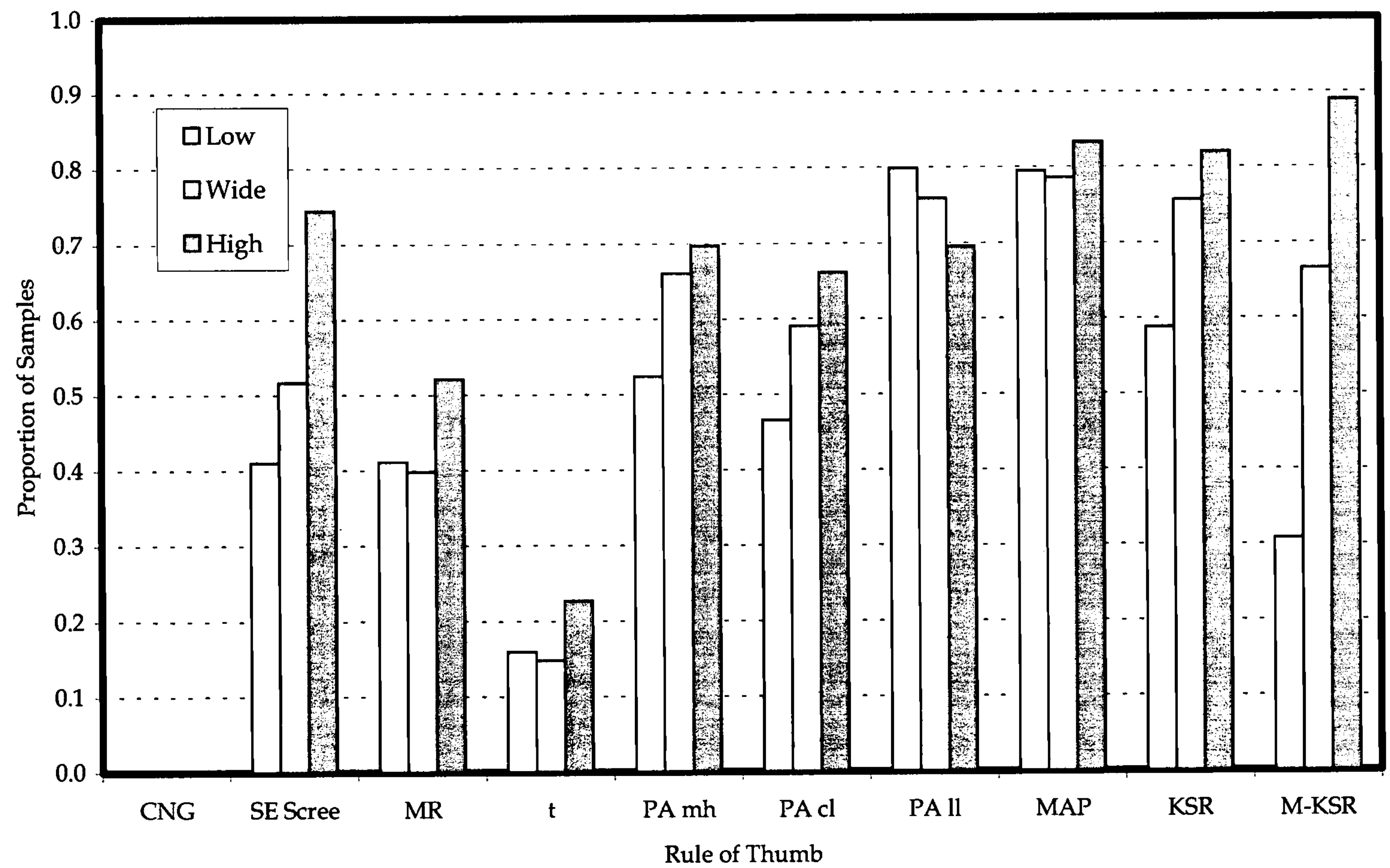
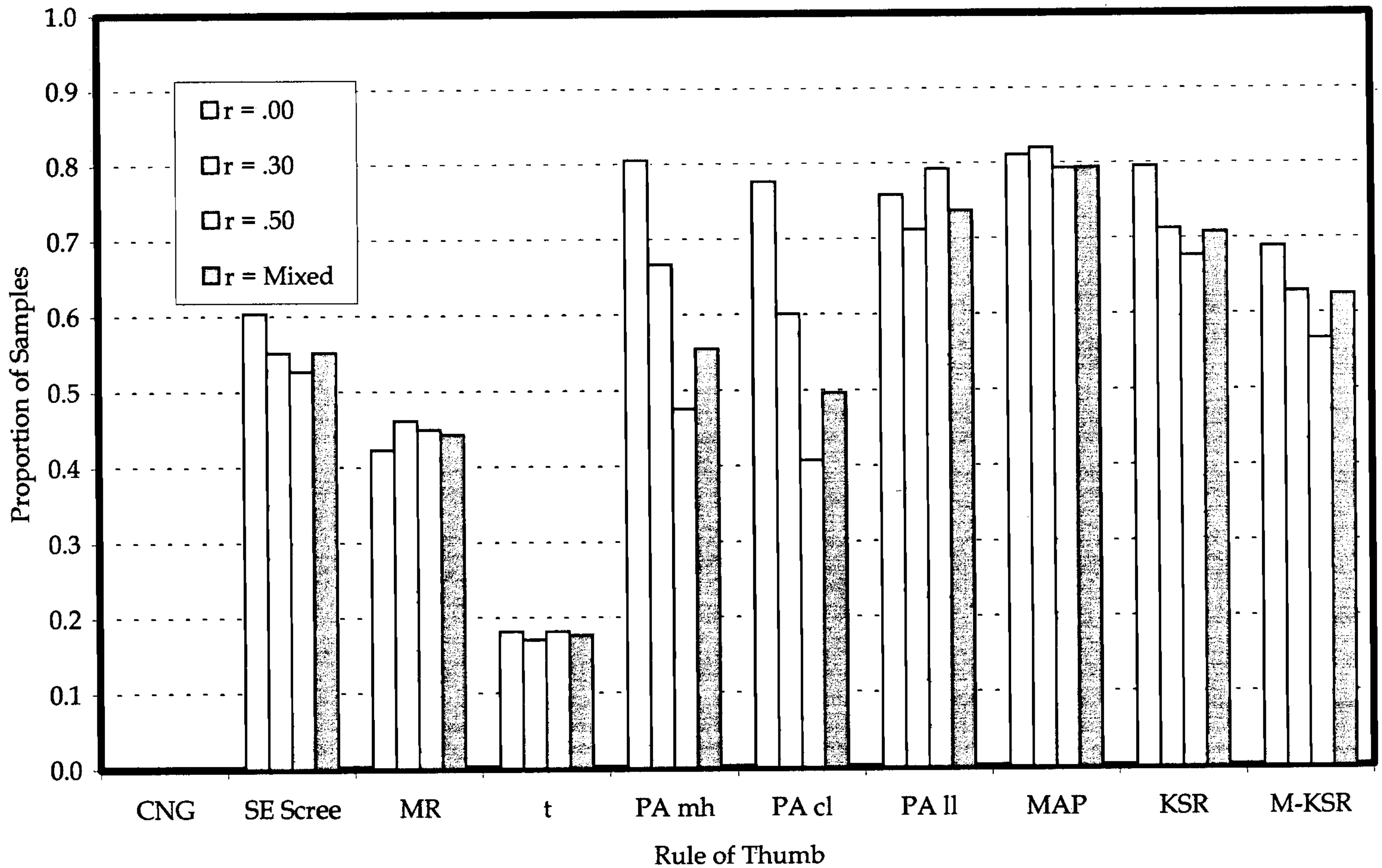


Figure 18. Proportion of Samples Agreeing with Rule Applied to Population R-Matrix by Phi



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